

**Nucleon transverse momentum-dependent parton distributions  
from domain wall fermion calculations at 297 MeV pion mass**

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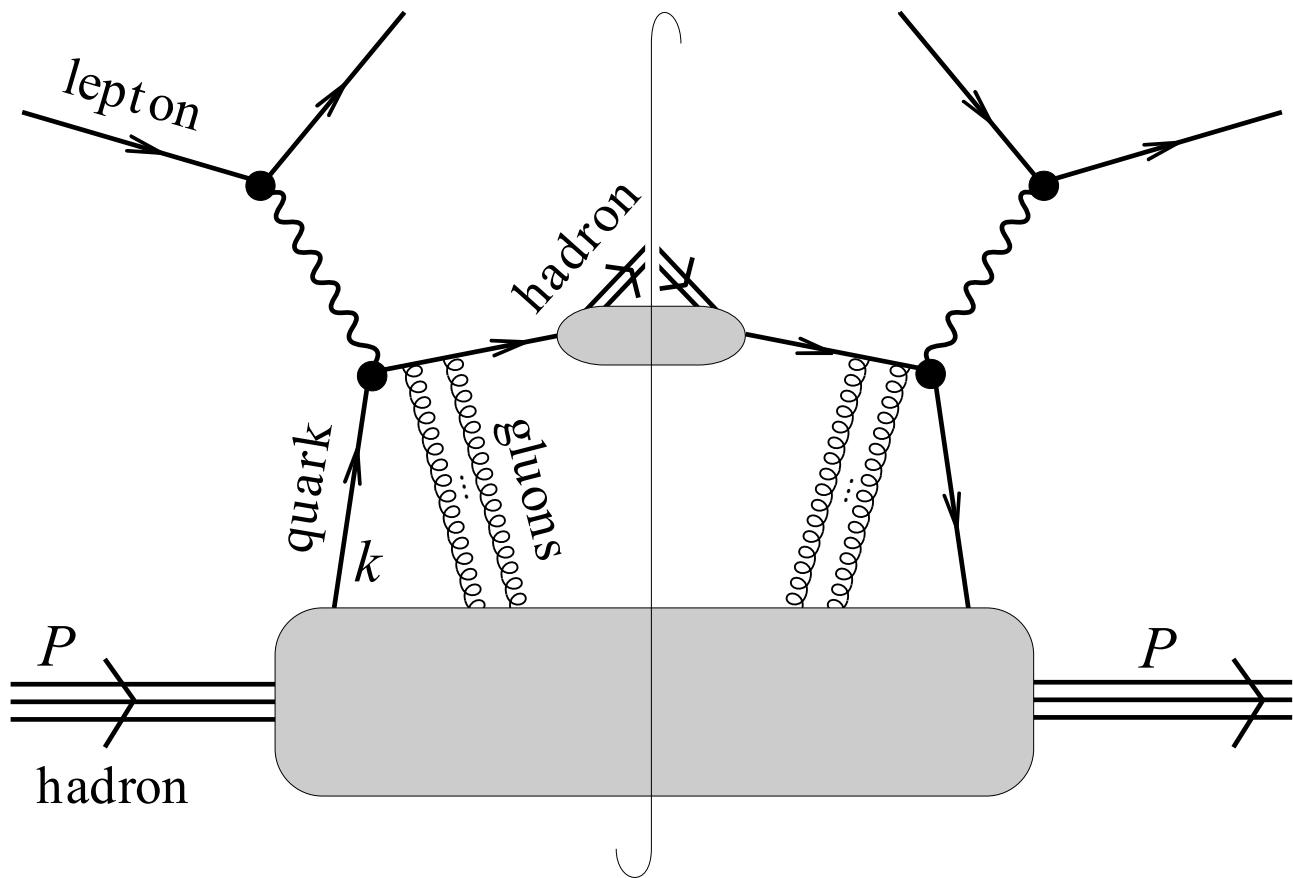
## Fundamental TMD correlator

$$\bar{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \dots) \equiv \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \dots, b] q(b) | P, S \rangle$$

$$\Phi^{[\Gamma]}(x, k_T, P, S, \dots) \equiv \int \frac{d^2 b_T}{(2\pi)^2} \int \frac{d(b \cdot P)}{(2\pi) P^+} \exp(ix(b \cdot P) - ib_T \cdot k_T) \left. \frac{\bar{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \dots)}{\bar{\mathcal{S}}(b^2, \dots)} \right|_{b^+=0}$$

- “Soft factor”  $\bar{\mathcal{S}}$  required to subtract divergences of Wilson line  $\mathcal{U}$
- $\bar{\mathcal{S}}$  is typically a combination of vacuum expectation values of Wilson line structures
- Here, will consider only ratios in which soft factors cancel

## Gauge link structure motivated by SIDIS

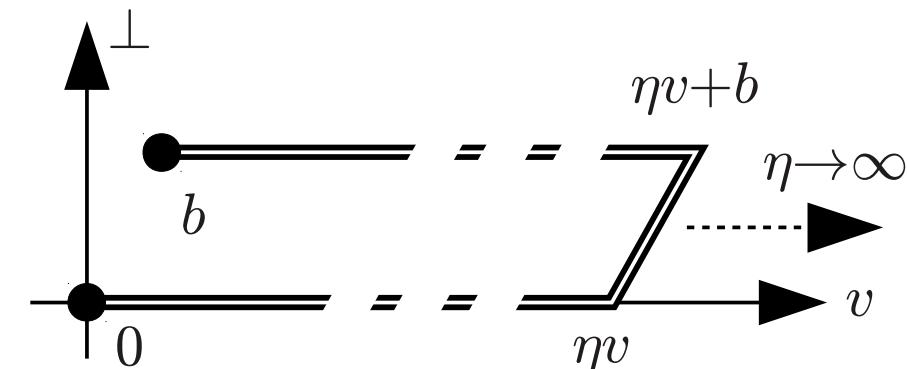


$$l + H(P) \longrightarrow l' + h(P_h) + X$$

Gauge link structure:

In matrix element  $\tilde{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \dots) \equiv \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \dots, b] q(b) | P, S \rangle$

Staple-shaped gauge link  $\mathcal{U}[0, \eta v, \eta v + b, b]$



incorporates SIDIS final state effects

## Gauge link structure motivated by SIDIS

Staple-shaped links incorporate SIDIS final state effects:

- Gauge link roughly follows direction of ejected quark, (close to) light cone
- Effective resummed description of gluon exchanges between ejected quark and remainder of nucleon in evolving final state
- Beyond tree level: Rapidity divergences suggest taking staple direction slightly off the light cone. Approach of Aybat, Collins, Qiu, Rogers makes  $v$  space-like. Parametrize in terms of Collins-Soper parameter

$$\hat{\zeta} \equiv \frac{P \cdot v}{|P||v|}$$

Light-like staple for  $\hat{\zeta} \rightarrow \infty$ . Perturbative evolution equations for large  $\hat{\zeta}$ .

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- Here, will consider only ratios in which soft factors cancel

## Decomposition of $\Phi$ into TMDs

All leading twist structures:

$$\Phi^{[\gamma^+]} = f_1 - \left[ \frac{\epsilon_{ij} k_i S_j}{m_H} f_{1T}^\perp \right]_{\text{odd}}$$

$$\Phi^{[\gamma^+ \gamma^5]} = \Lambda g_1 + \frac{k_T \cdot S_T}{m_H} g_{1T}$$

$$\Phi^{[i\sigma^{i+} \gamma^5]} = S_i h_1 + \frac{(2k_i k_j - k_T^2 \delta_{ij}) S_j}{2m_H^2} h_{1T}^\perp + \frac{\Lambda k_i}{m_H} h_{1L}^\perp + \left[ \frac{\epsilon_{ij} k_j}{m_H} h_1^\perp \right]_{\text{odd}}$$

## TMD Classification

All leading twist structures:

H ↓	$q \rightarrow$	U	L	T
U	$f_1$			$h_1^\perp$
L			$g_1$	$h_{1L}^\perp$
T		$f_{1T}^\perp$	$g_{1T}$	$h_1$ $h_{1T}^\perp$

↑  
Sivers (T-odd)

← Boer-Mulders  
(T-odd)

## Decomposition of $\widetilde{\Phi}$ into amplitudes

$$\widetilde{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \hat{\zeta}, \mu) \equiv \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \eta v, \eta v + b, b] q(b) | P, S \rangle$$

Decompose in terms of invariant amplitudes; at leading twist,

$$\frac{1}{2P^+} \widetilde{\Phi}_{\text{unsubtr.}}^{[\gamma^+]} = \bar{A}_{2B} + im_H \epsilon_{ij} b_i S_j \bar{A}_{12B}$$

$$\frac{1}{2P^+} \widetilde{\Phi}_{\text{unsubtr.}}^{[\gamma^+ \gamma^5]} = -\Lambda \bar{A}_{6B} + i[(b \cdot P)\Lambda - m_H(b_T \cdot S_T)] \bar{A}_{7B}$$

$$\begin{aligned} \frac{1}{2P^+} \widetilde{\Phi}_{\text{unsubtr.}}^{[i\sigma^{i+} \gamma^5]} &= im_H \epsilon_{ij} b_j \bar{A}_{4B} - S_i \bar{A}_{9B} \\ &\quad - im_H \Lambda b_i \bar{A}_{10B} + m_H[(b \cdot P)\Lambda - m_H(b_T \cdot S_T)] b_i \bar{A}_{11B} \end{aligned}$$

## Fourier-transformed TMDs

$$\tilde{f}(x, b_T^2, \dots) \equiv \int d^2 k_T \exp(ib_T \cdot k_T) f(x, k_T^2, \dots)$$

$$\tilde{f}^{(n)}(x, b_T^2, \dots) \equiv n! \left( -\frac{2}{m_H^2} \partial_{b_T^2} \right)^n \tilde{f}(x, b_T^2, \dots)$$

In limit  $|b_T| \rightarrow 0$ , recover  $k_T$ -moments:

$$\tilde{f}^{(n)}(x, 0, \dots) \equiv \int d^2 k_T \left( \frac{k_T^2}{2m_H^2} \right)^n f(x, k_T^2, \dots) \equiv f^{(n)}(x)$$

ill-defined for large  $k_T$ , so will not attempt to extrapolate to  $b_T = 0$ , but give results at finite  $|b_T|$ .

In this study, only consider first  $x$ -moments (accessible at  $b \cdot P = 0$ ), rather than scanning range of  $b \cdot P$ :

$$f^{[1]}(k_T^2, \dots) \equiv \int_{-1}^1 dx f(x, k_T^2, \dots)$$

## Relation between Fourier-transformed TMDs and invariant amplitudes $\bar{A}_i$

Invariant amplitudes directly give selected  $x$ -integrated TMDs in Fourier ( $b_T$ ) space (showing just the ones relevant for Sivers, Boer-Mulders shifts), up to soft factors:

$$\tilde{f}_1^{[1](0)}(b_T^2, \hat{\zeta}, \dots, \eta v \cdot P) = 2\bar{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\bar{S}(b^2, \dots)$$

$$\tilde{f}_{1T}^{\perp[1](1)}(b_T^2, \hat{\zeta}, \dots, \eta v \cdot P) = -2\bar{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\bar{S}(b^2, \dots)$$

$$\tilde{h}_1^{\perp[1](1)}(b_T^2, \hat{\zeta}, \dots, \eta v \cdot P) = 2\bar{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\bar{S}(b^2, \dots)$$

## Generalized shifts

Form ratios in which soft factors, ( $\Gamma$ -independent) multiplicative renormalization factors cancel

Boer-Mulders shift:

$$\langle k_y \rangle_{UT} \equiv m_H \frac{\tilde{h}_1^{\perp[1](1)}}{\tilde{f}_1^{[1](0)}} = \left. \frac{\int dx \int d^2 k_T k_y \Phi[\gamma^+ + s^j i\sigma^j + \gamma^5](x, k_T, P, \dots)}{\int dx \int d^2 k_T \Phi[\gamma^+ + s^j i\sigma^j + \gamma^5](x, k_T, P, \dots)} \right|_{s_T=(1,0)}$$

Average transverse momentum of quarks polarized in the orthogonal transverse (“ $T$ ”) direction in an unpolarized (“ $U$ ”) hadron; normalized to the number of valence quarks. “Dipole moment” in  $b_T^2 = 0$  limit, “shift”.

**Issue:**  $k_T$ -moments in this ratio singular; generalize to ratio of Fourier-transformed TMDs at *nonzero*  $b_T^2$ ,

$$\langle k_y \rangle_{UT}(b_T^2, \dots) \equiv m_H \frac{\tilde{h}_1^{\perp[1](1)}(b_T^2, \dots)}{\tilde{f}_1^{[1](0)}(b_T^2, \dots)}$$

(remember singular  $b_T \rightarrow 0$  limit corresponds to taking  $k_T$ -moment). “Generalized shift”.

## Generalized shifts from amplitudes

Now, can also express this in terms of invariant amplitudes:

$$\langle k_y \rangle_{UT}(b_T^2, \dots) \equiv m_H \frac{\tilde{h}_1^{\perp[1](1)}(b_T^2, \dots)}{\tilde{f}_1^{[1](0)}(b_T^2, \dots)} = m_H \frac{\bar{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\bar{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Analogously, Sivers shift (in a polarized hadron):

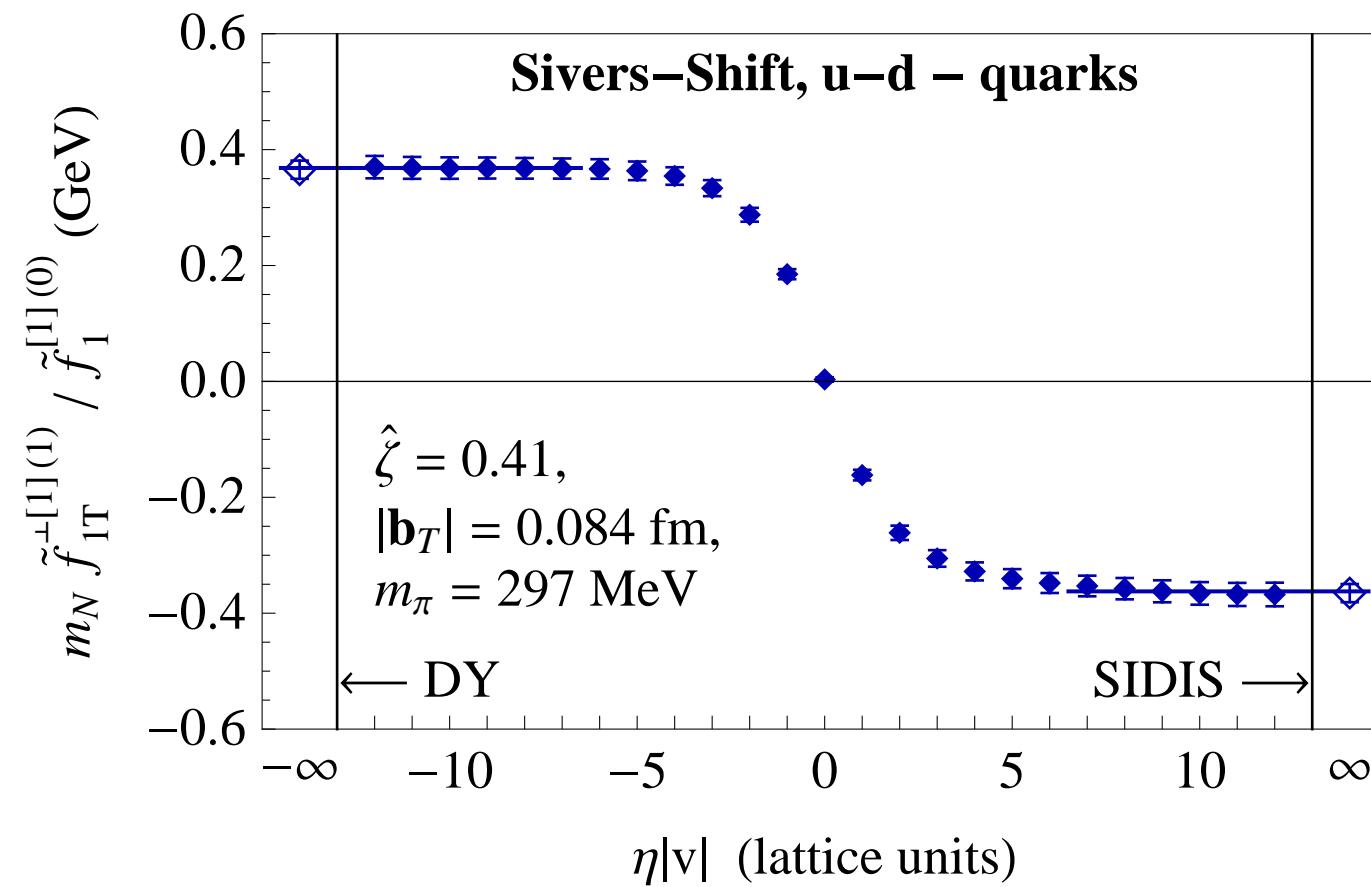
$$\langle k_y \rangle_{TU}(b_T^2, \dots) = -m_H \frac{\bar{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\bar{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

## Lattice setup

- Evaluate directly  $\widetilde{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \hat{\zeta}, \mu) \equiv \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \eta v, \eta v + b, b] q(b) | P, S \rangle$
- Euclidean time: Place entire operator at one time slice, i.e.,  $b, \eta v$  purely spatial
- Since generic  $b, v$  space-like, no obstacle to boosting system to such a frame!
- Parametrization of correlator in terms of  $\bar{A}_i$  invariants permits direct translation of results back to original frame
- Form desired ratios of  $\bar{A}_i$  invariants
- Extrapolate  $\eta \rightarrow \infty, \hat{\zeta} \rightarrow \infty$  numerically.
- Here: Use RBC/UKQCD 2+1-flavor gauge ensemble with  $a \approx 0.084$  fm,  $m_\pi = 297$  MeV; compare with previous mixed action DWF on MILC ensemble calculation at  $m_\pi = 518$  MeV with  $a \approx 0.12$  fm.
- Use variety of  $P, b, \eta v$ ; here  $b \perp P, b \perp v$  (lowest  $x$ -moment, kinematical choices/constraints)

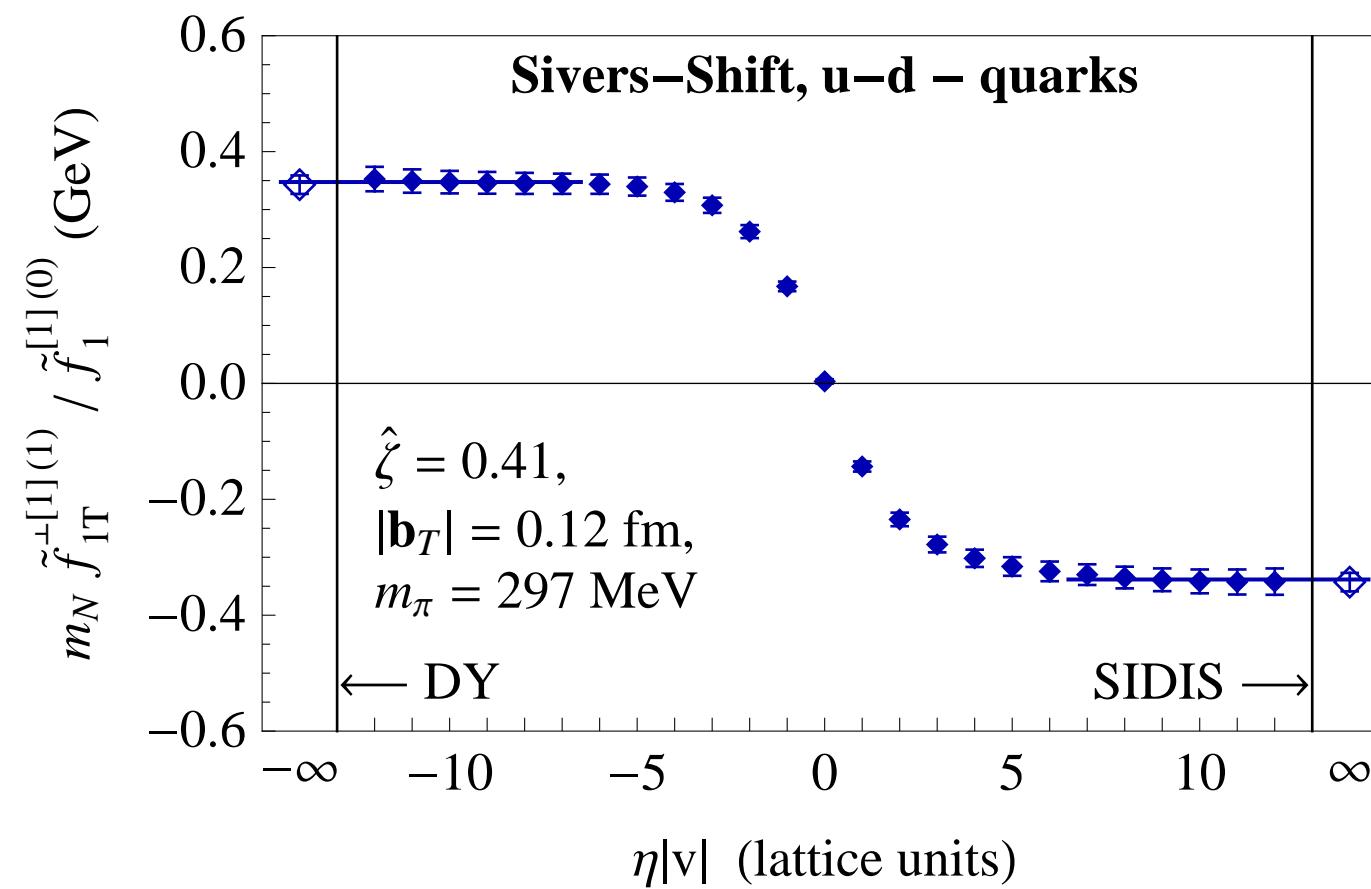
## Results: Sivers shift

Dependence on staple extent; sequence of panels at different  $|b_T|$



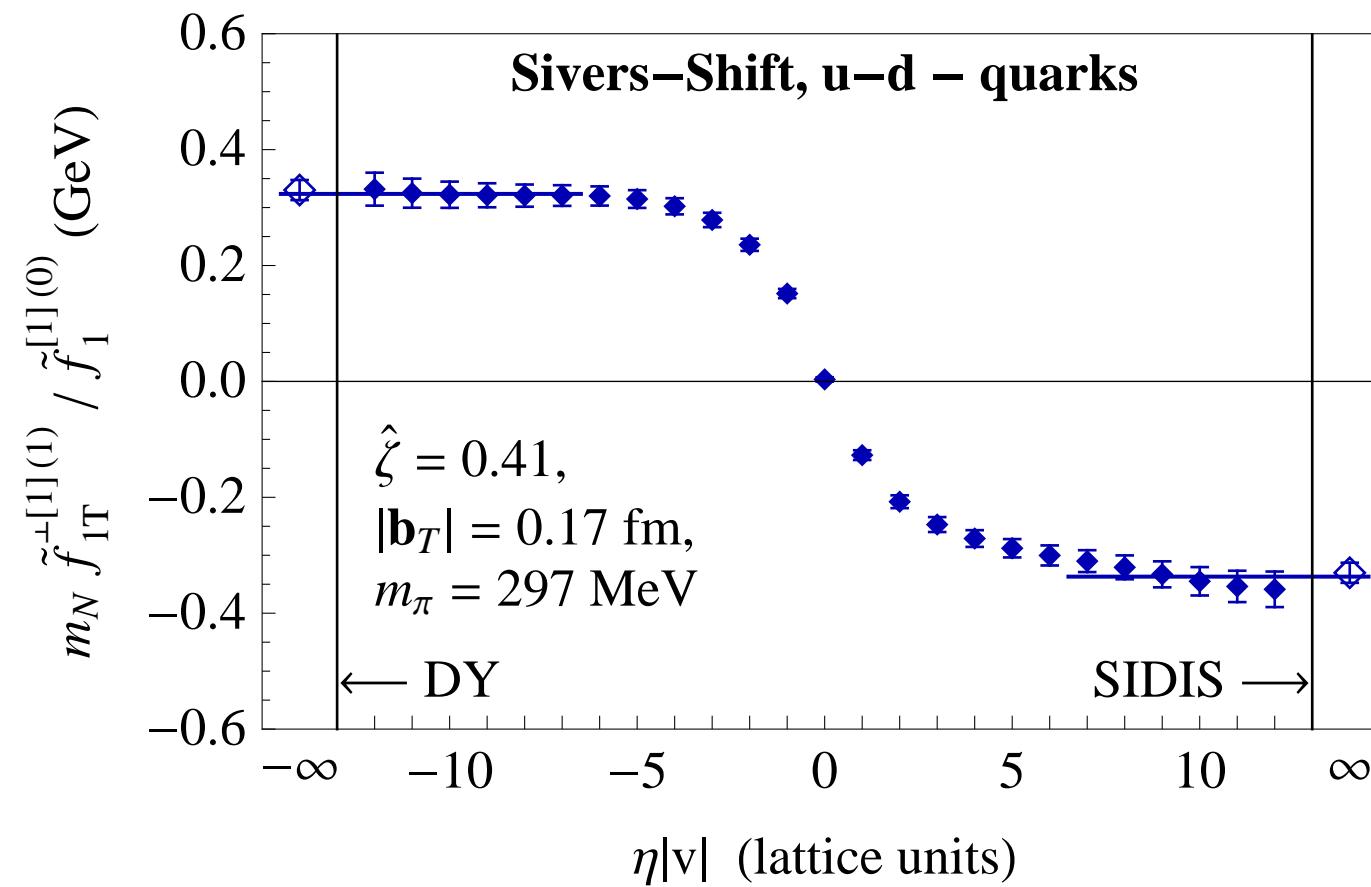
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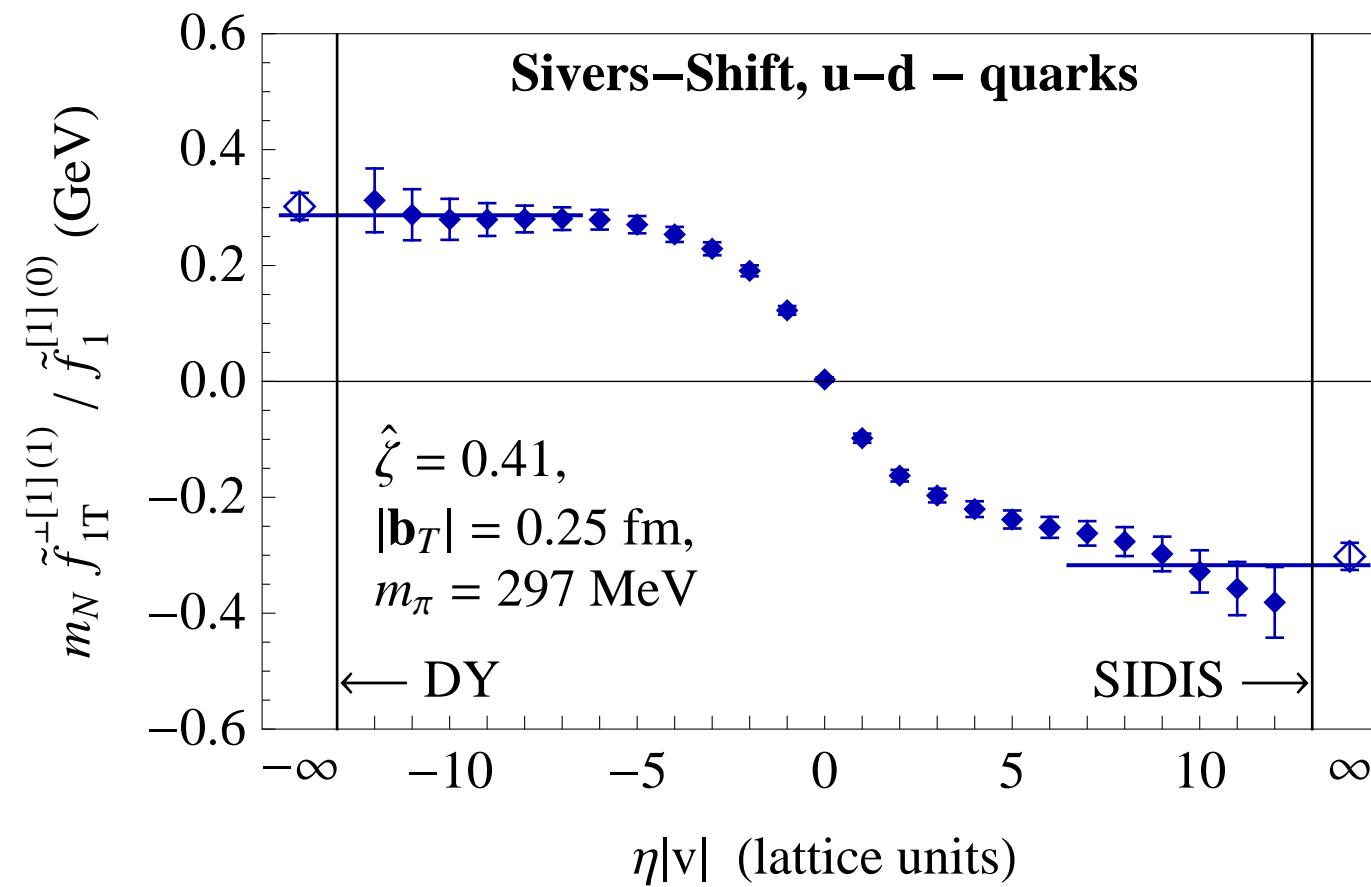
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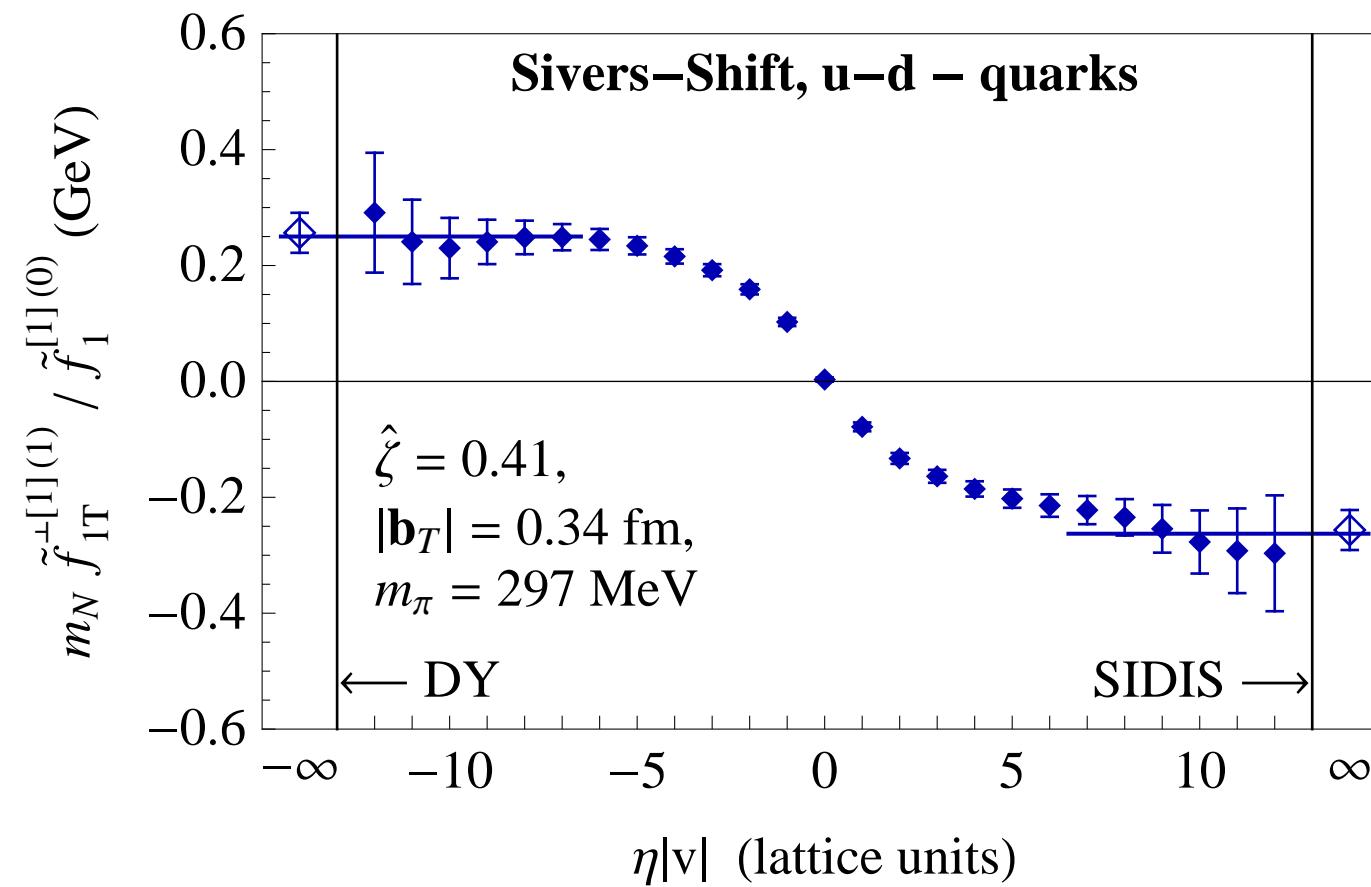
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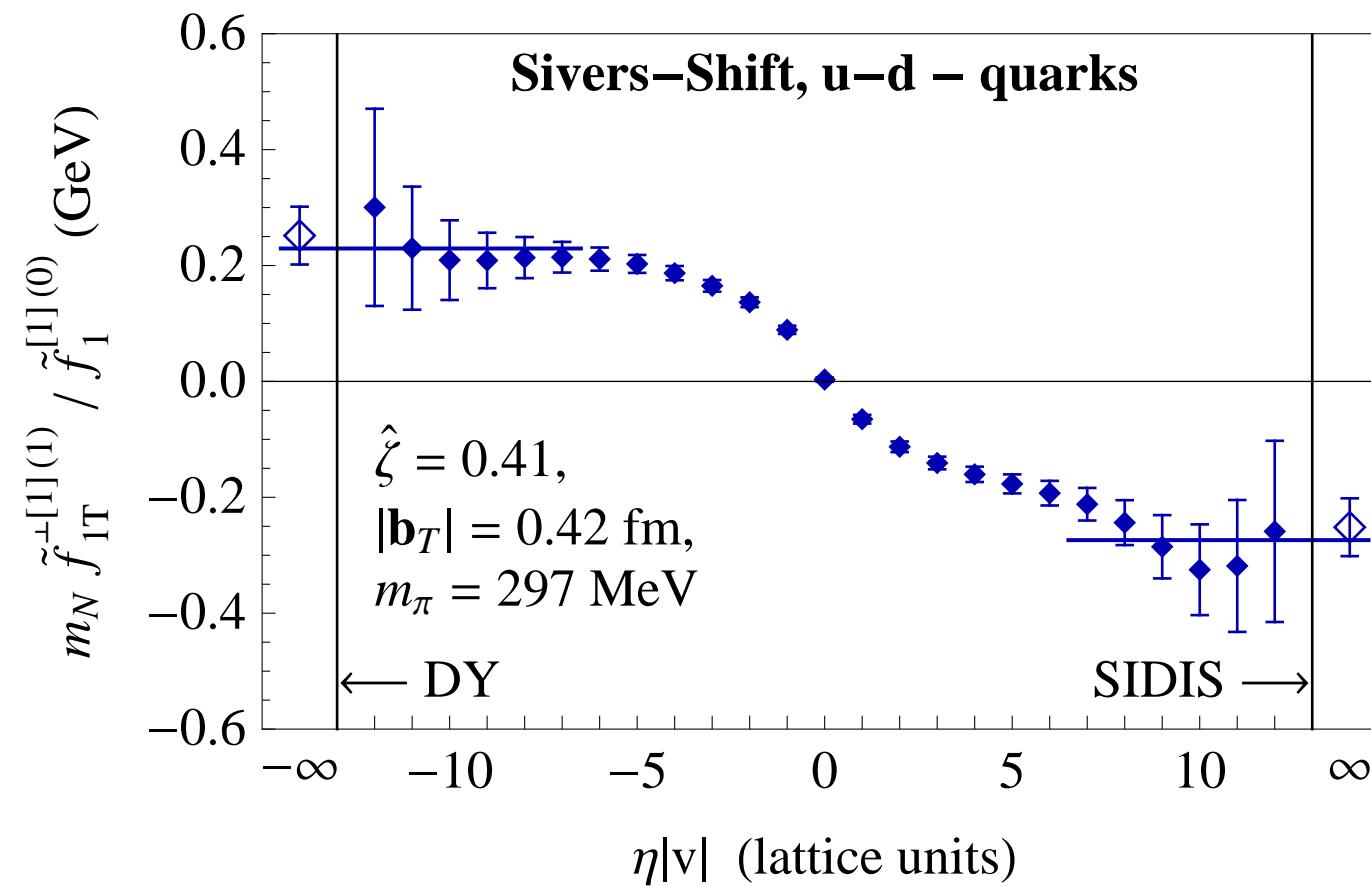
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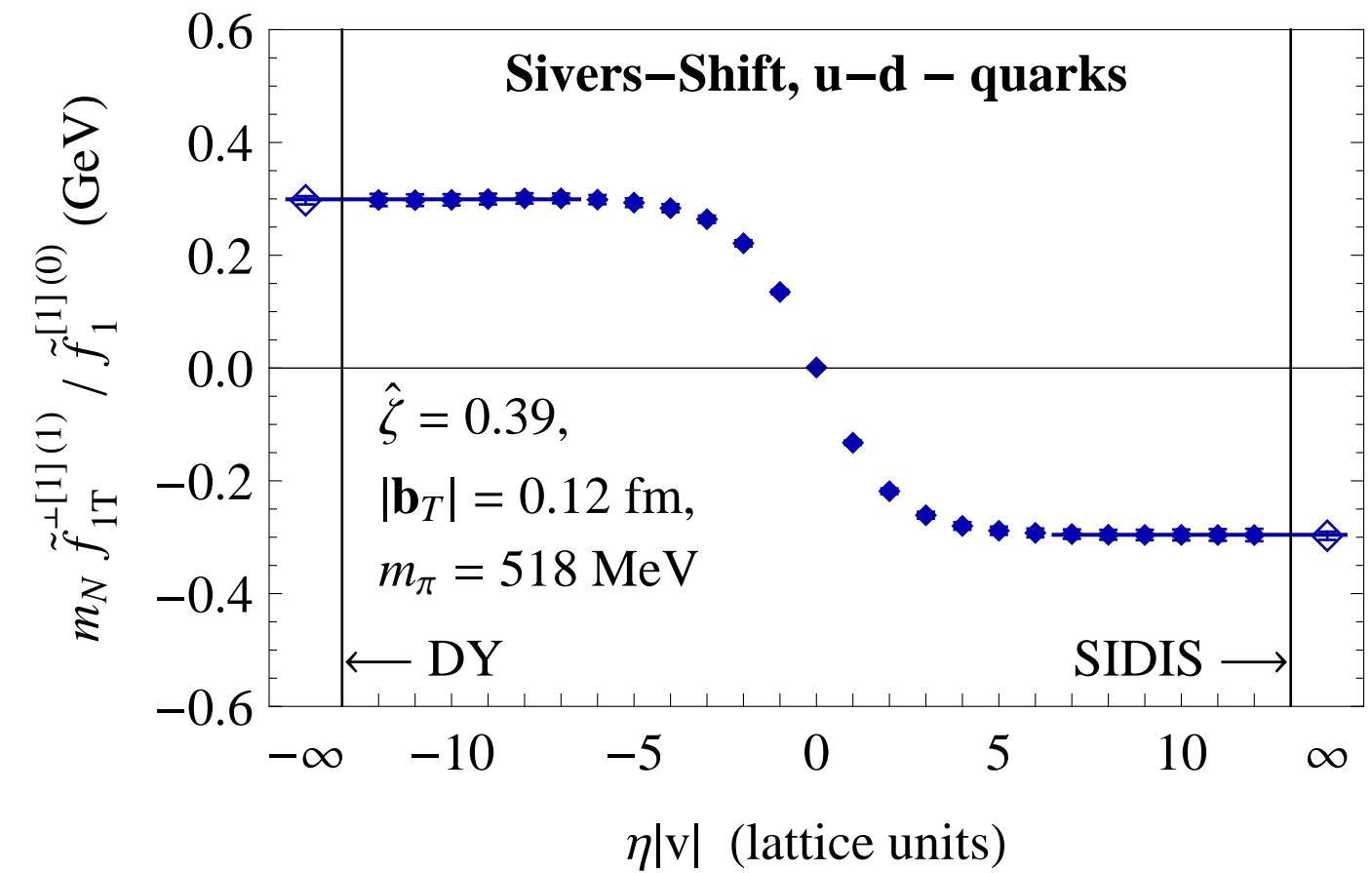
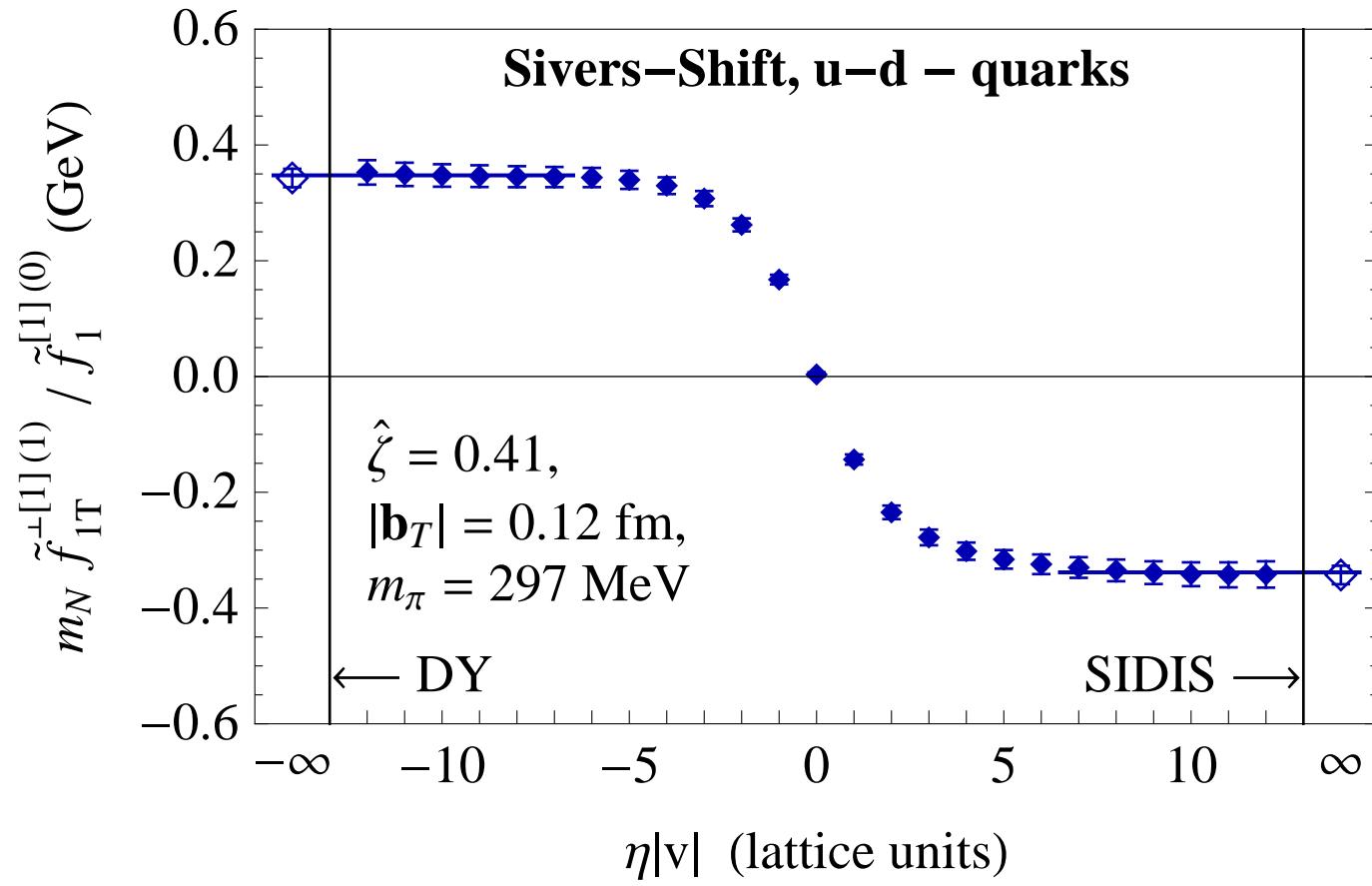
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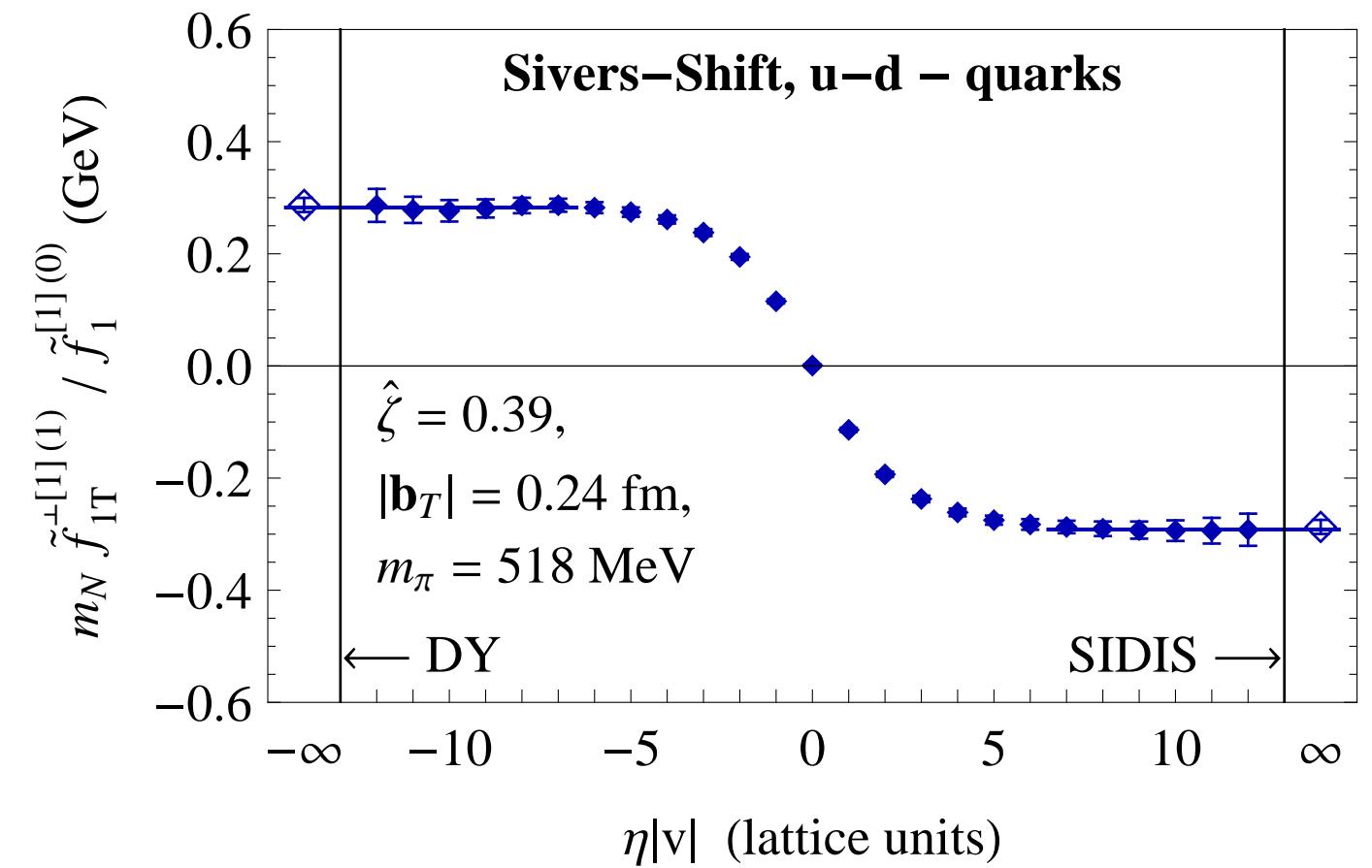
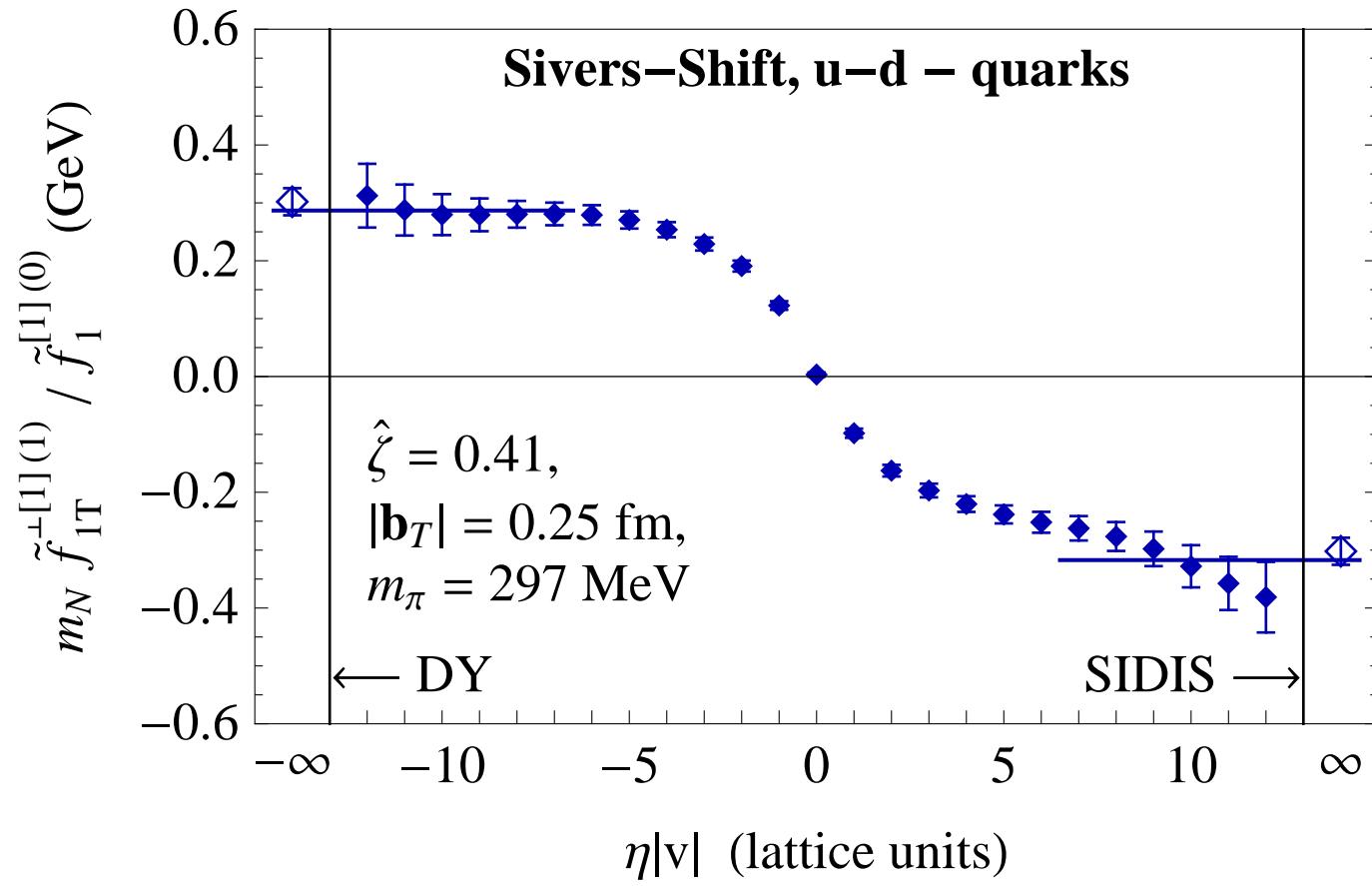
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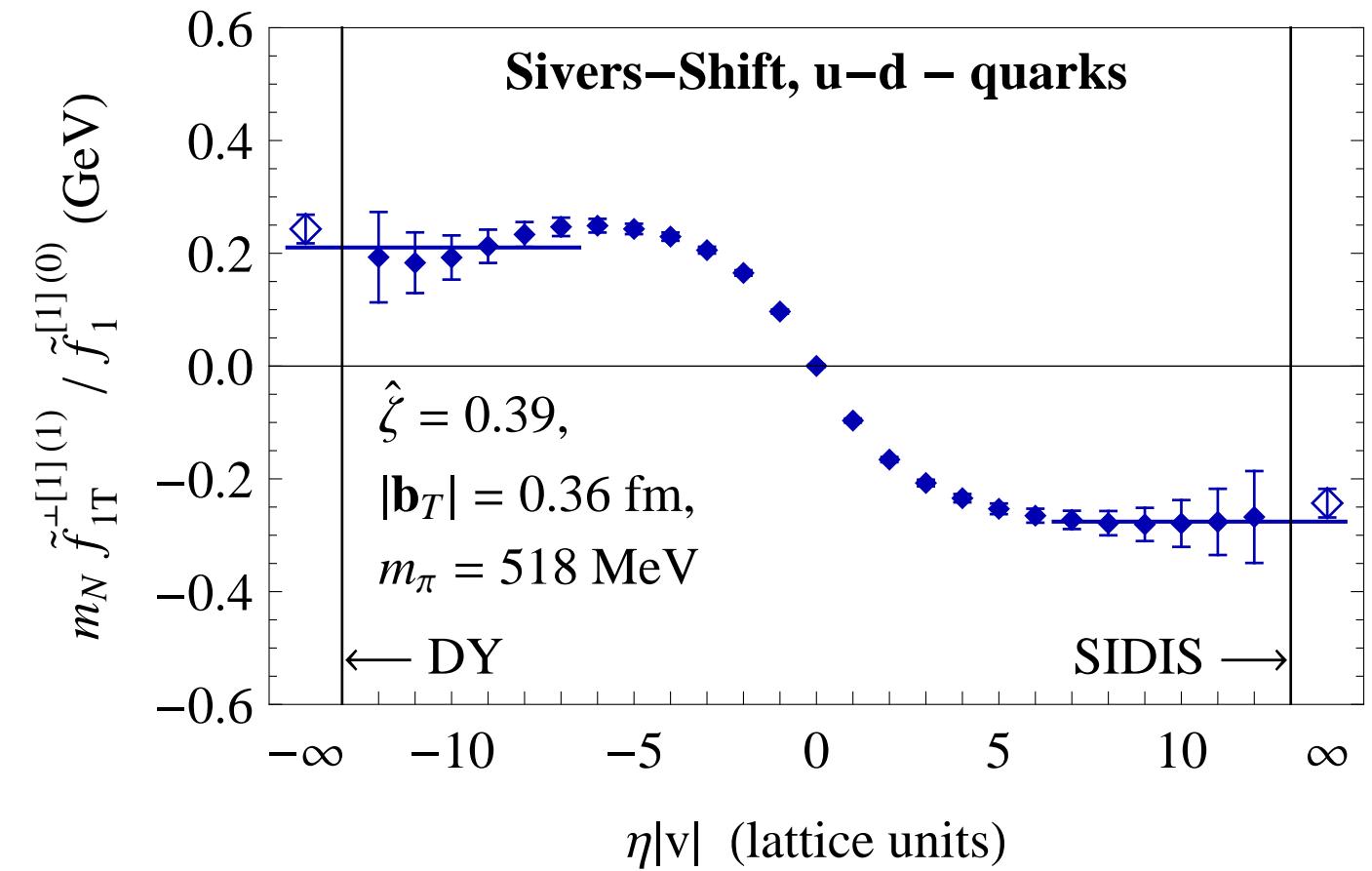
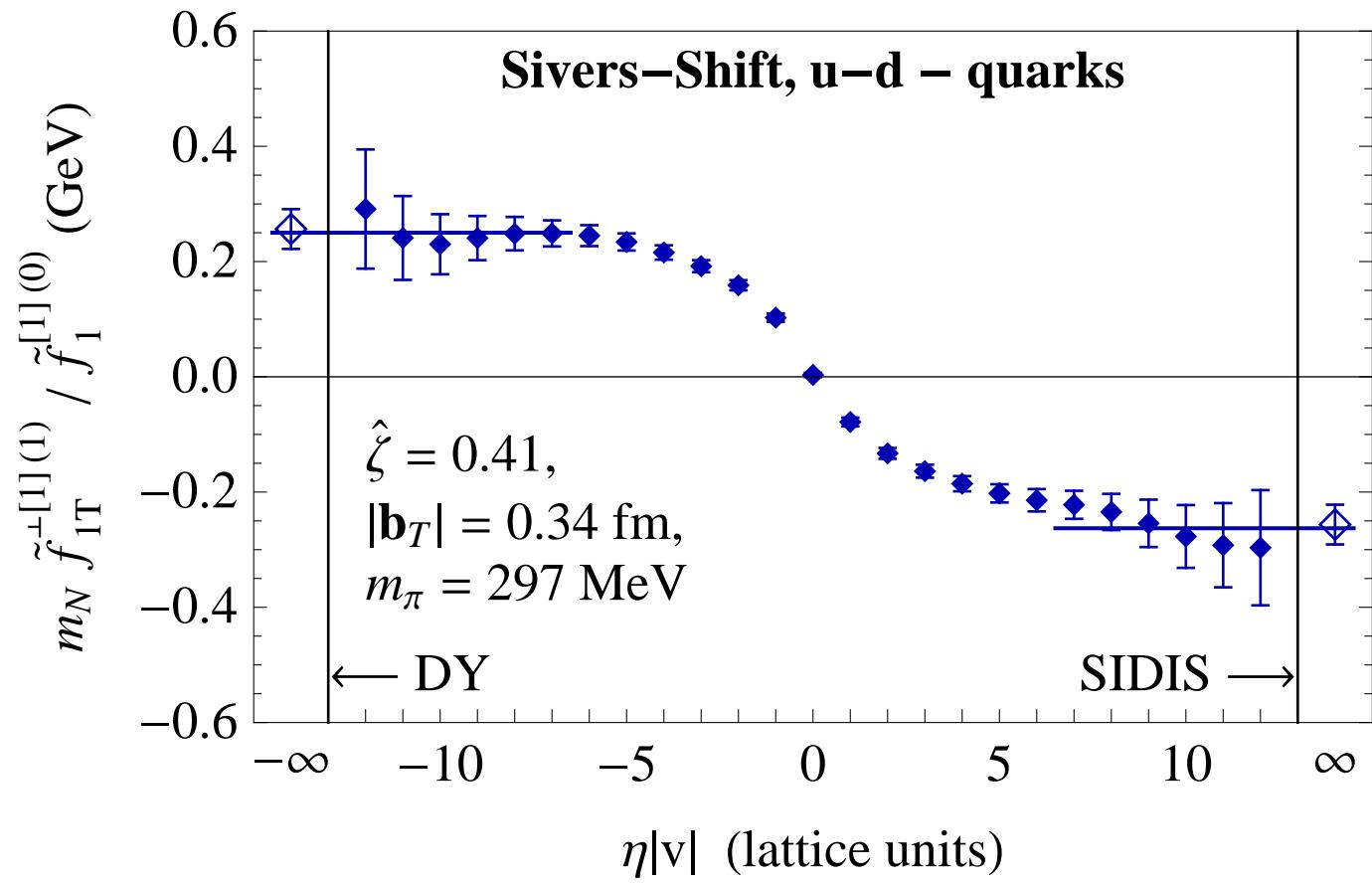
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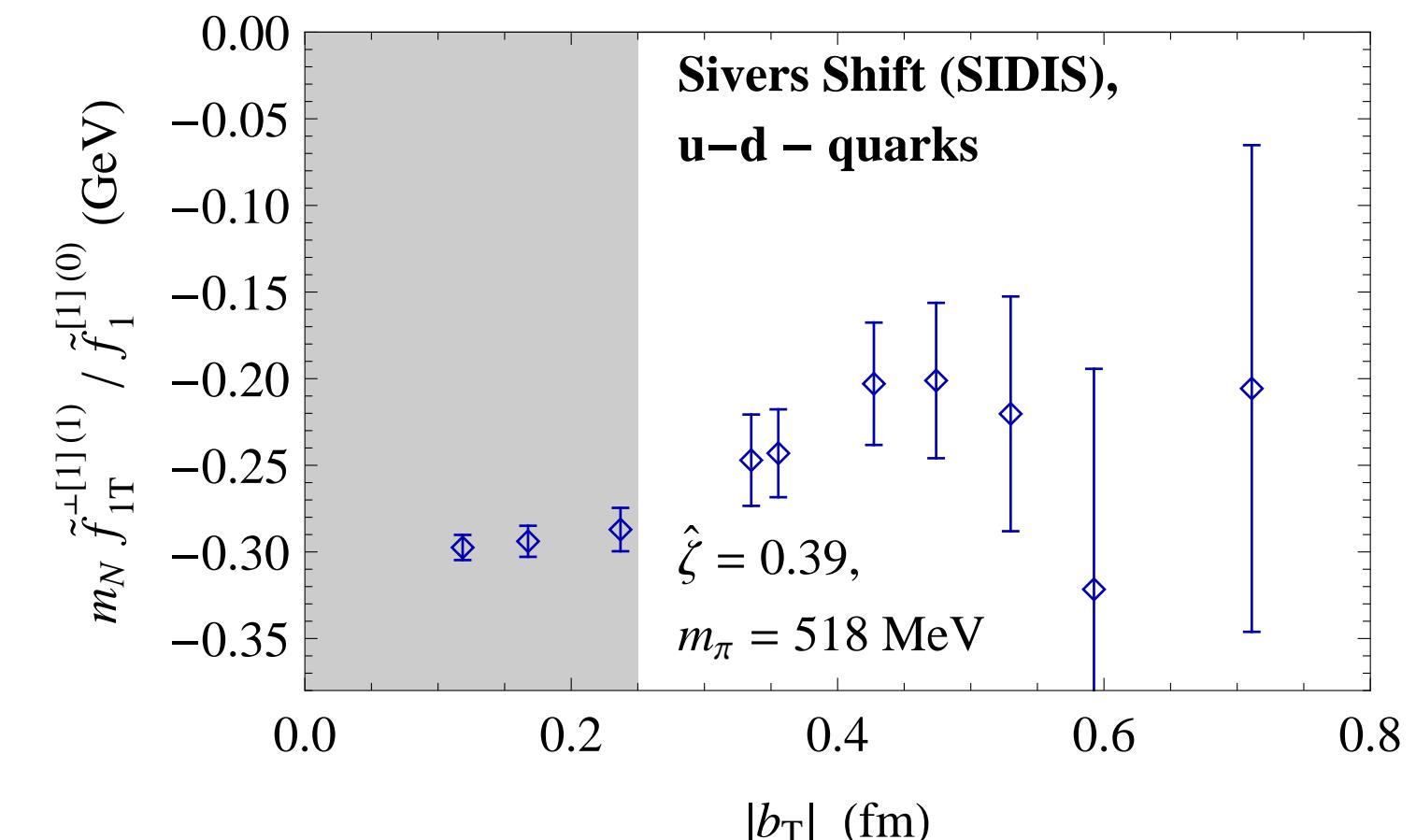
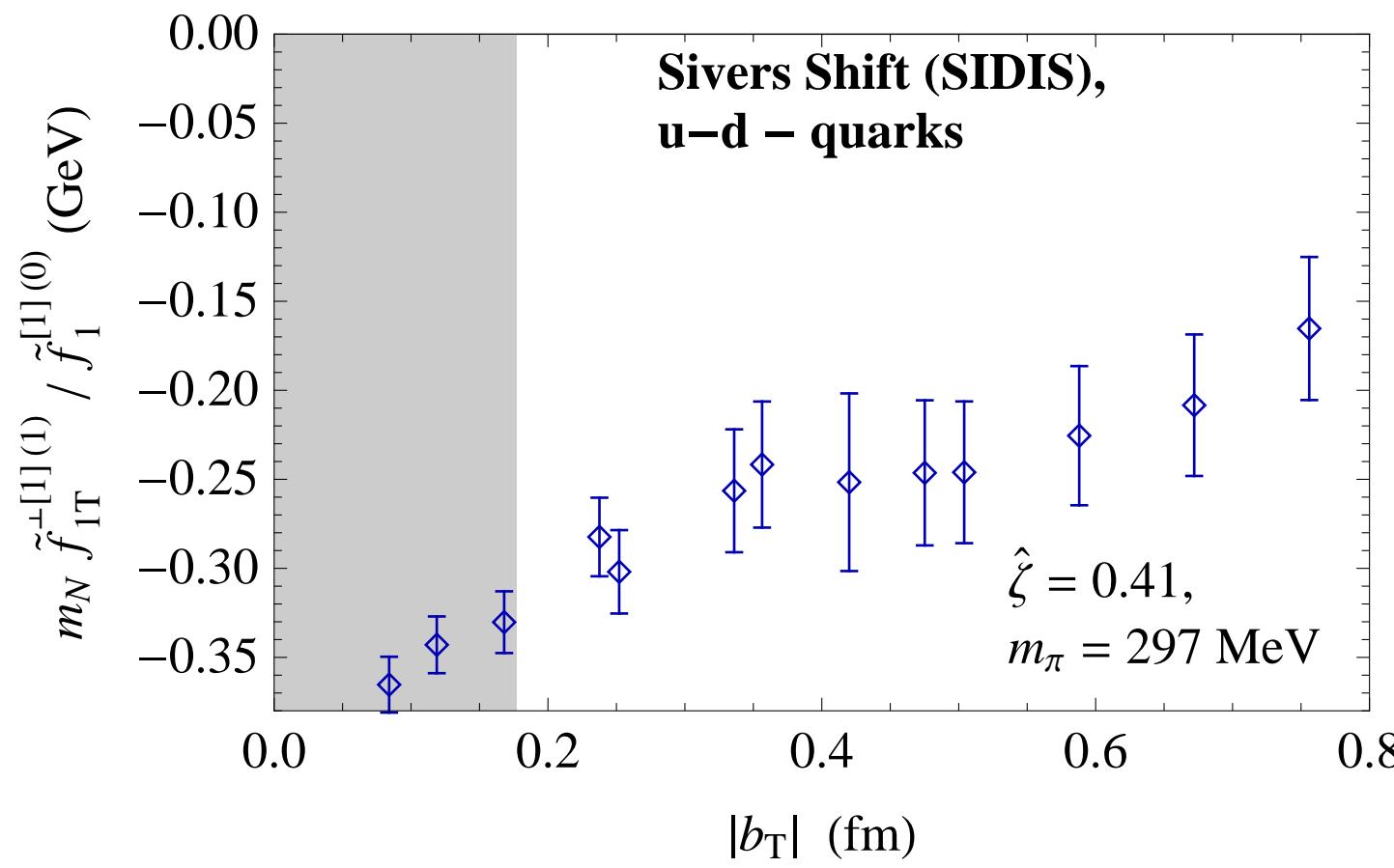
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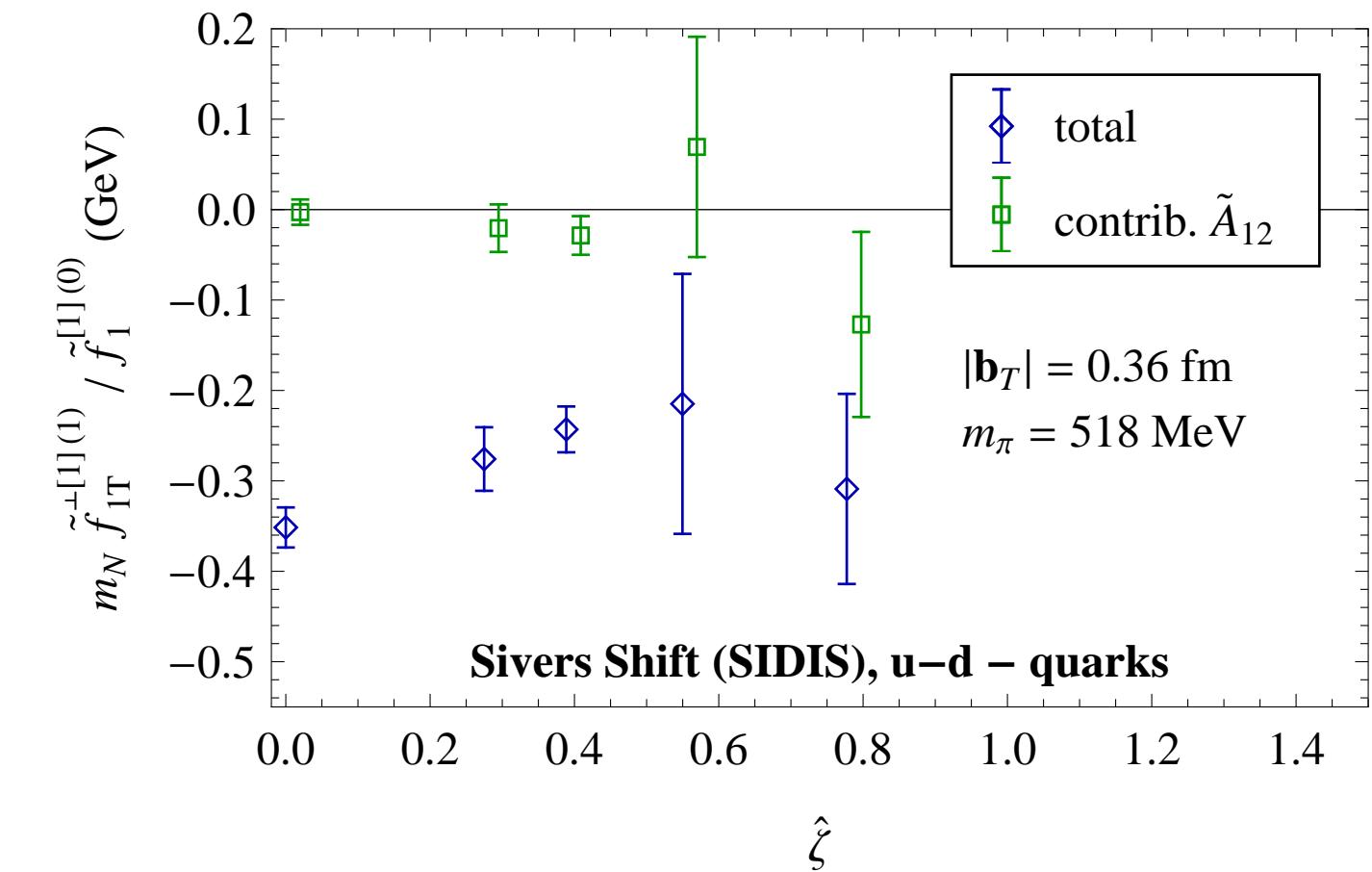
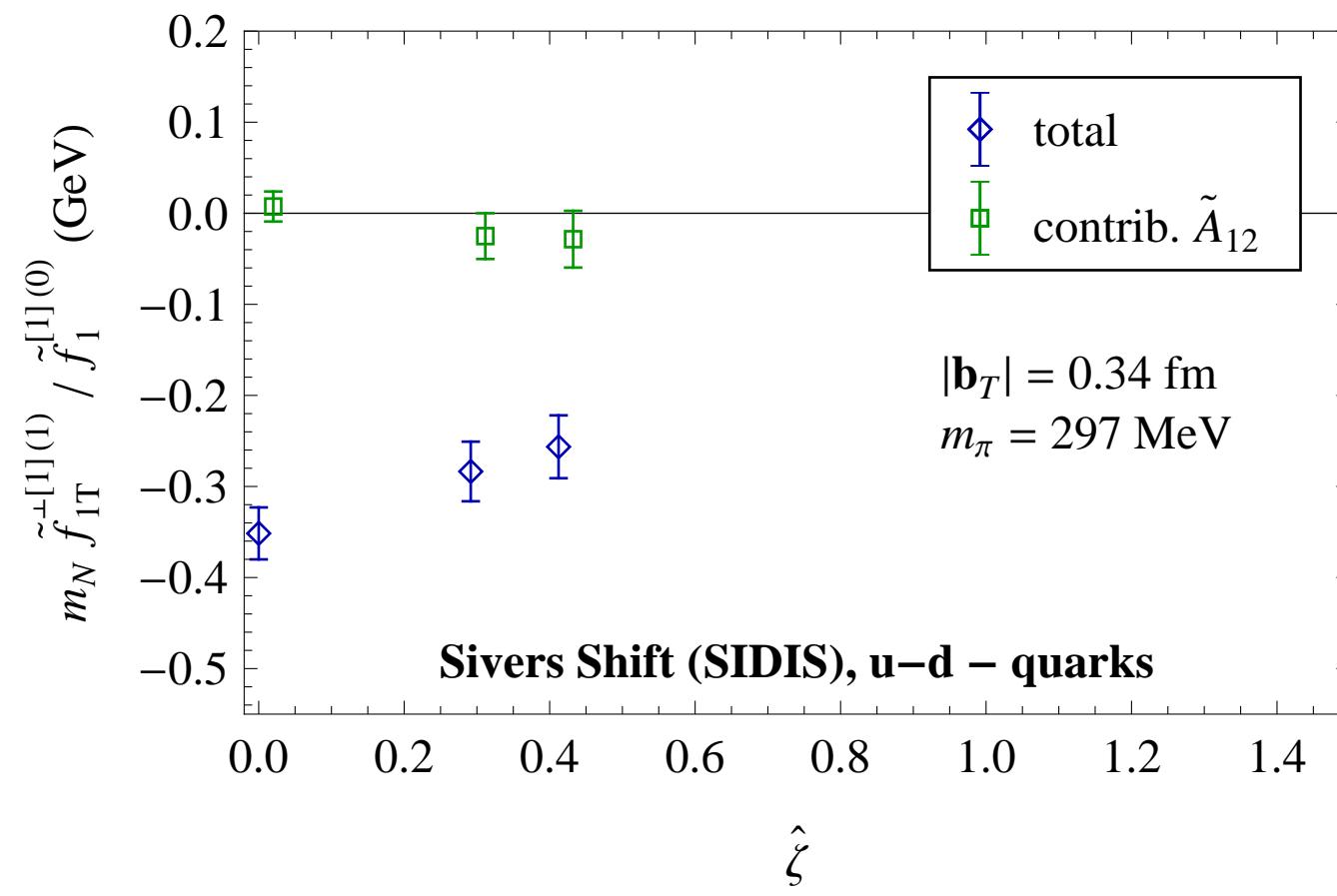
## Results: Sivers shift

Dependence of SIDIS limit on  $|b_T|$



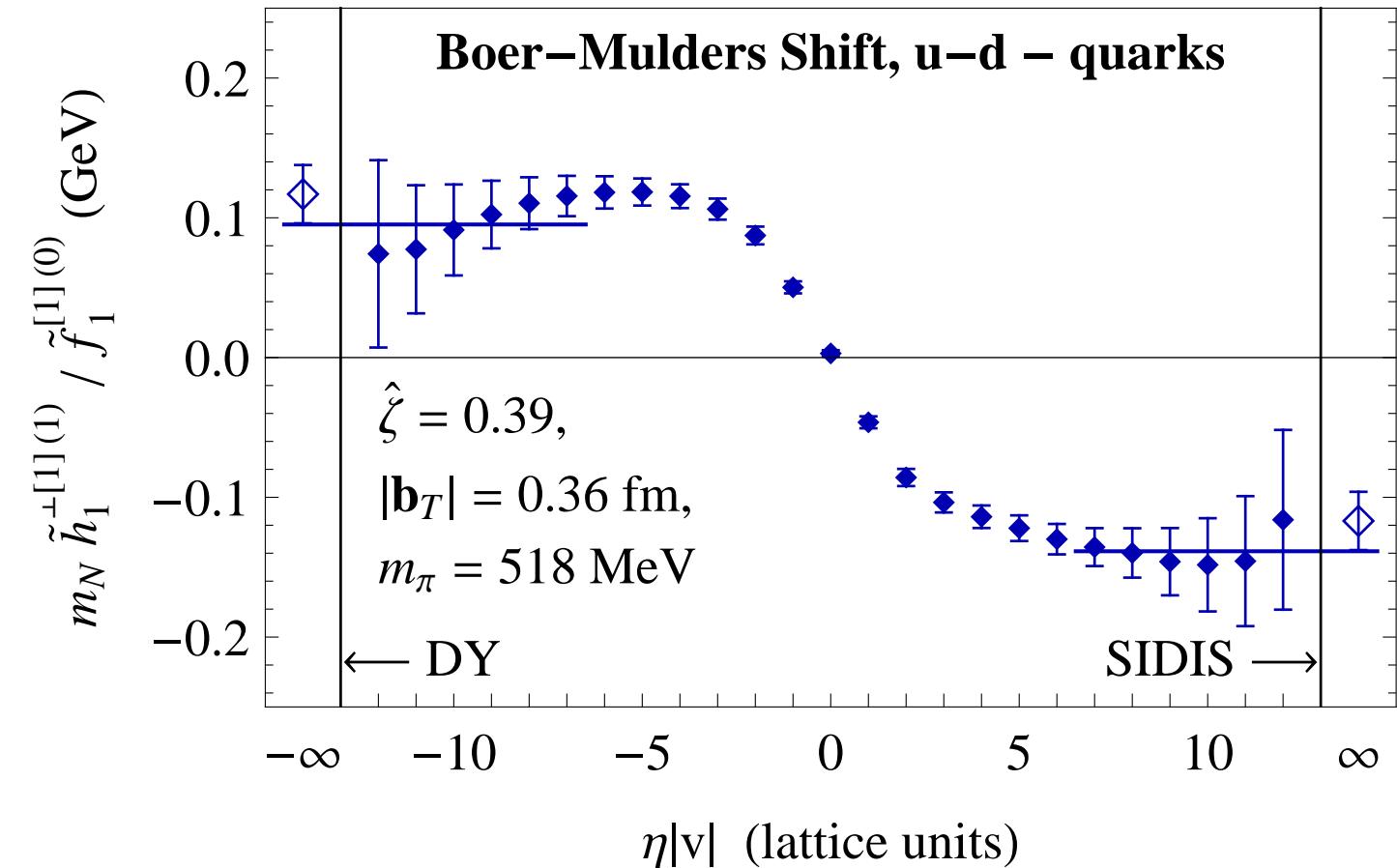
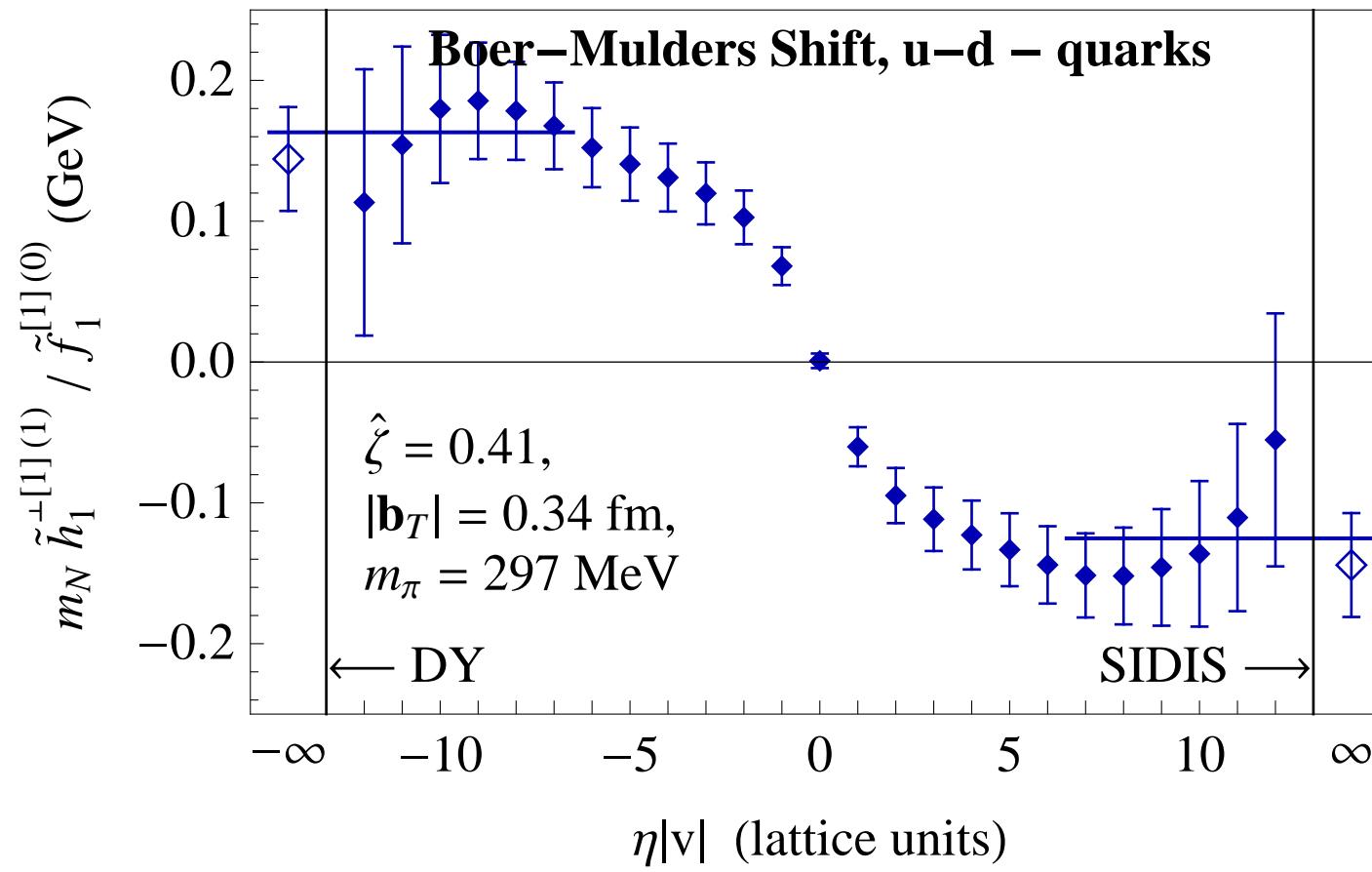
## Results: Sivers shift

Dependence of SIDIS limit on  $\hat{\zeta}$



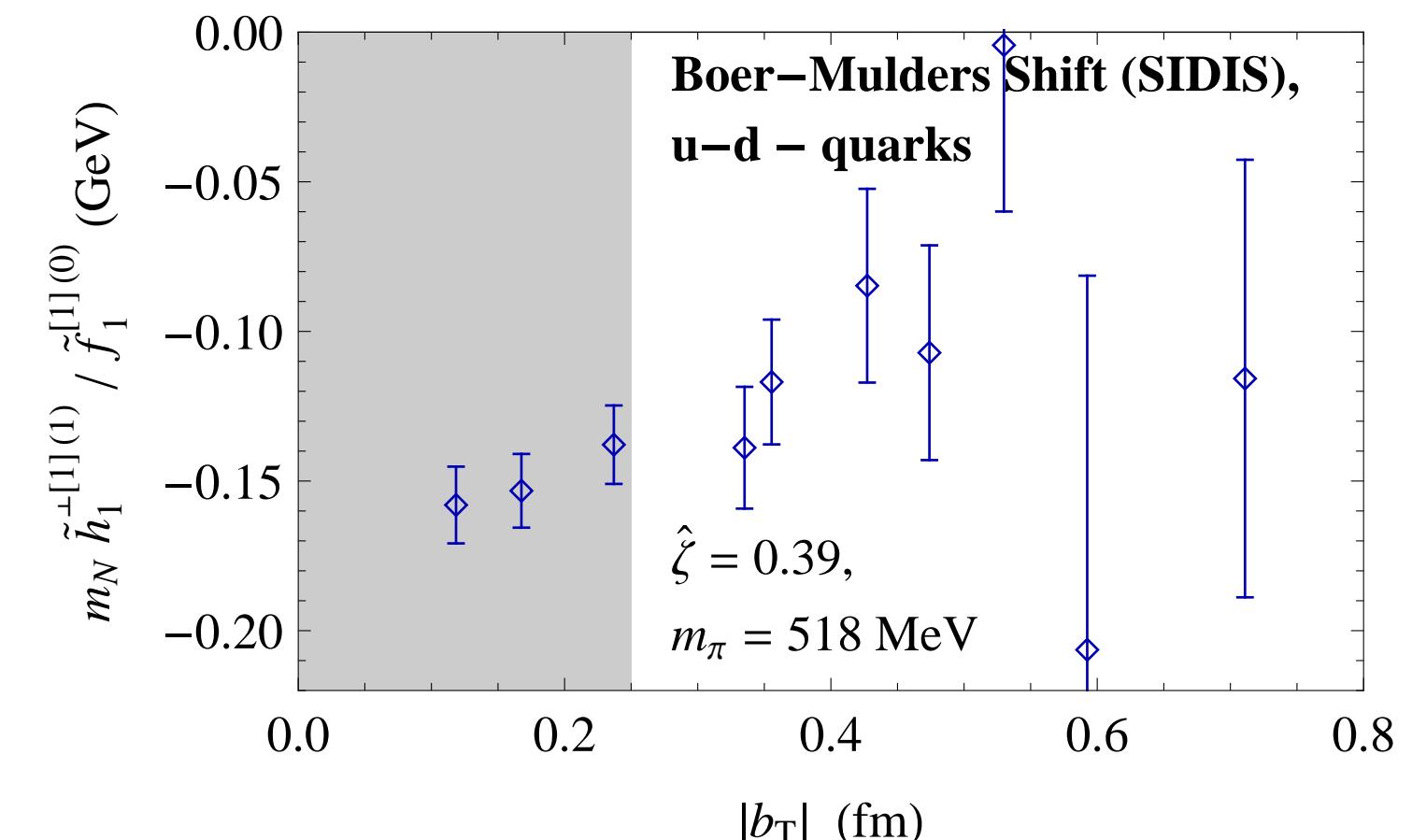
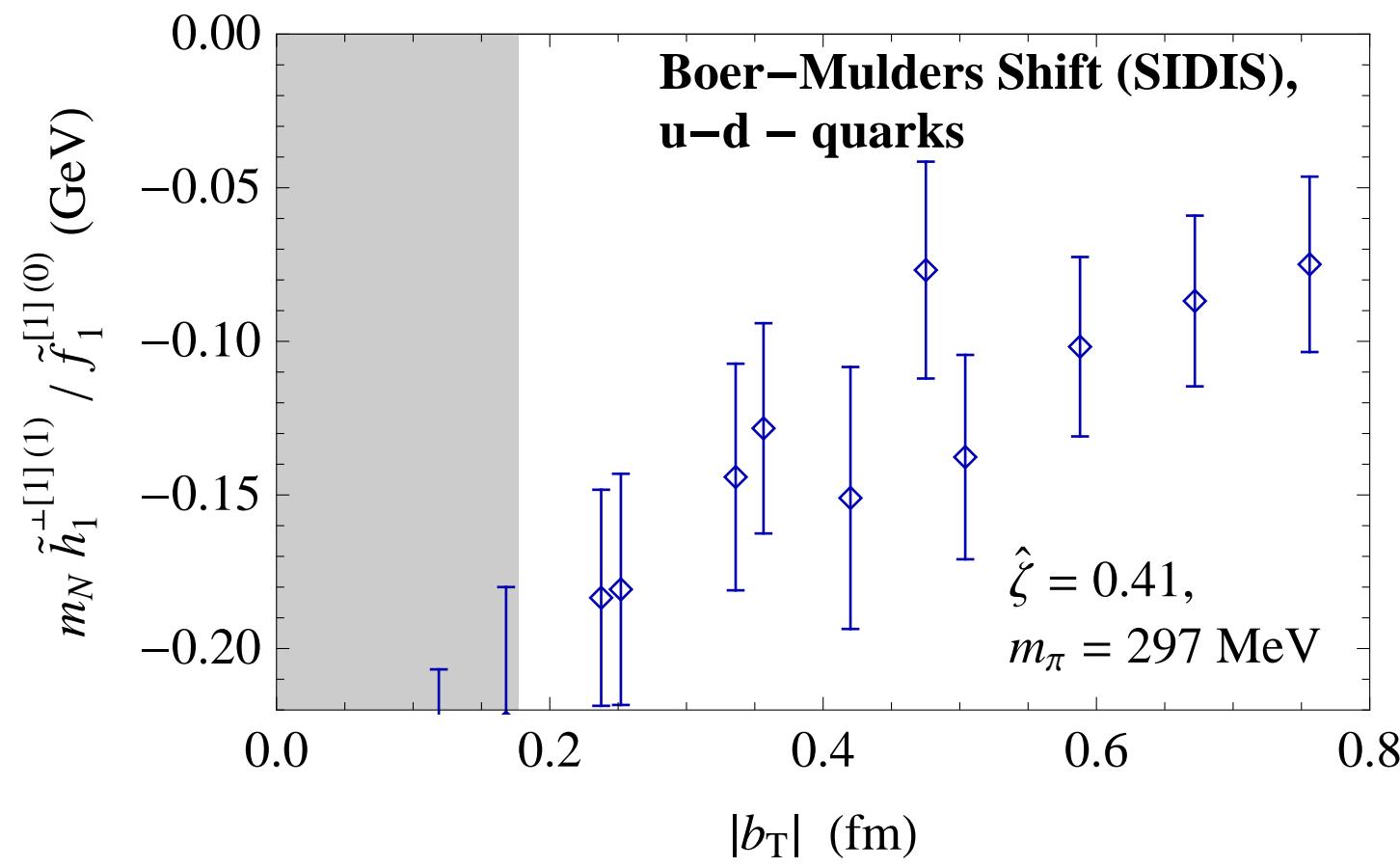
## Results: Boer-Mulders shift

Dependence on staple extent



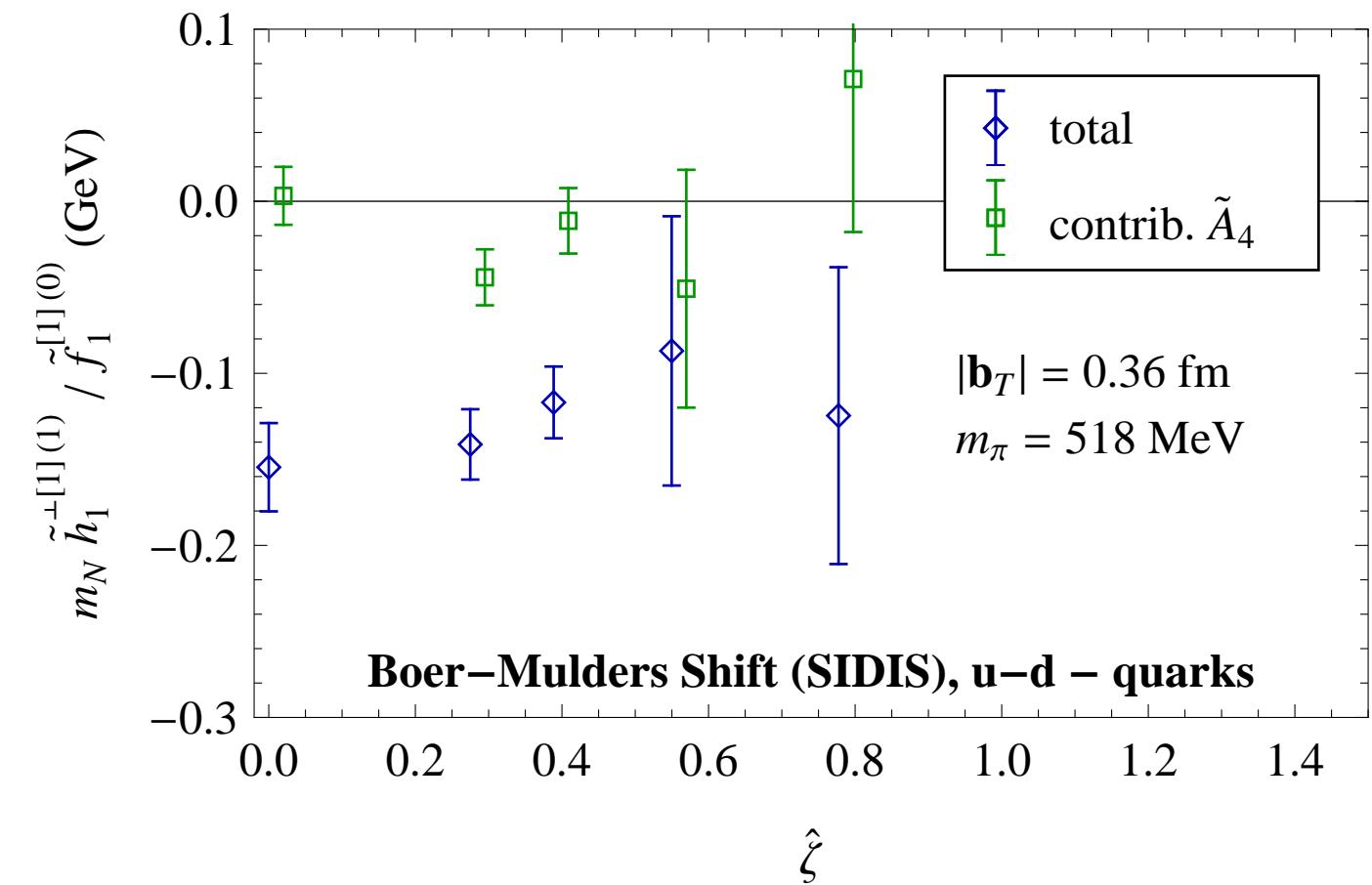
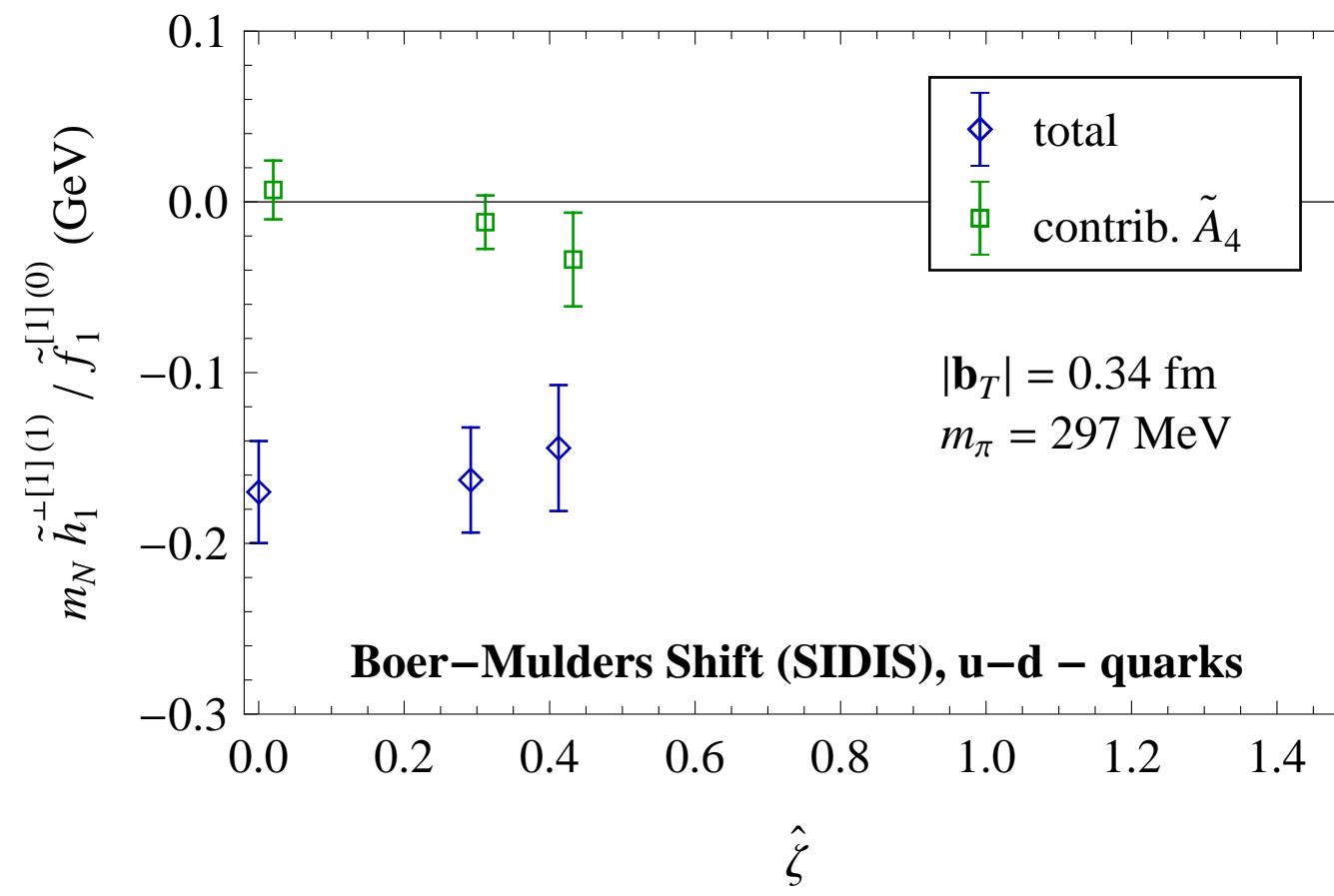
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Dependence of SIDIS limit on  $|b_T|$



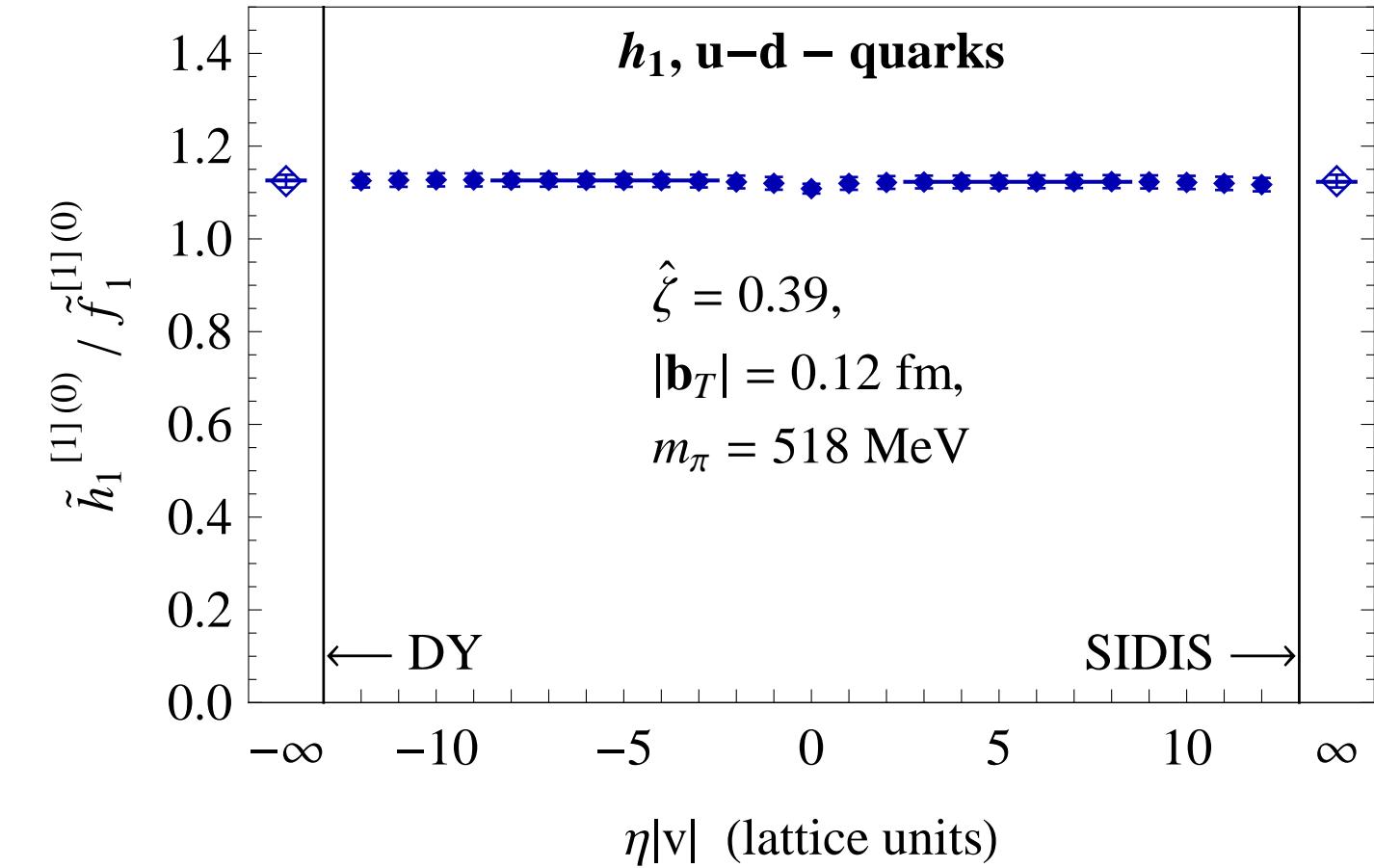
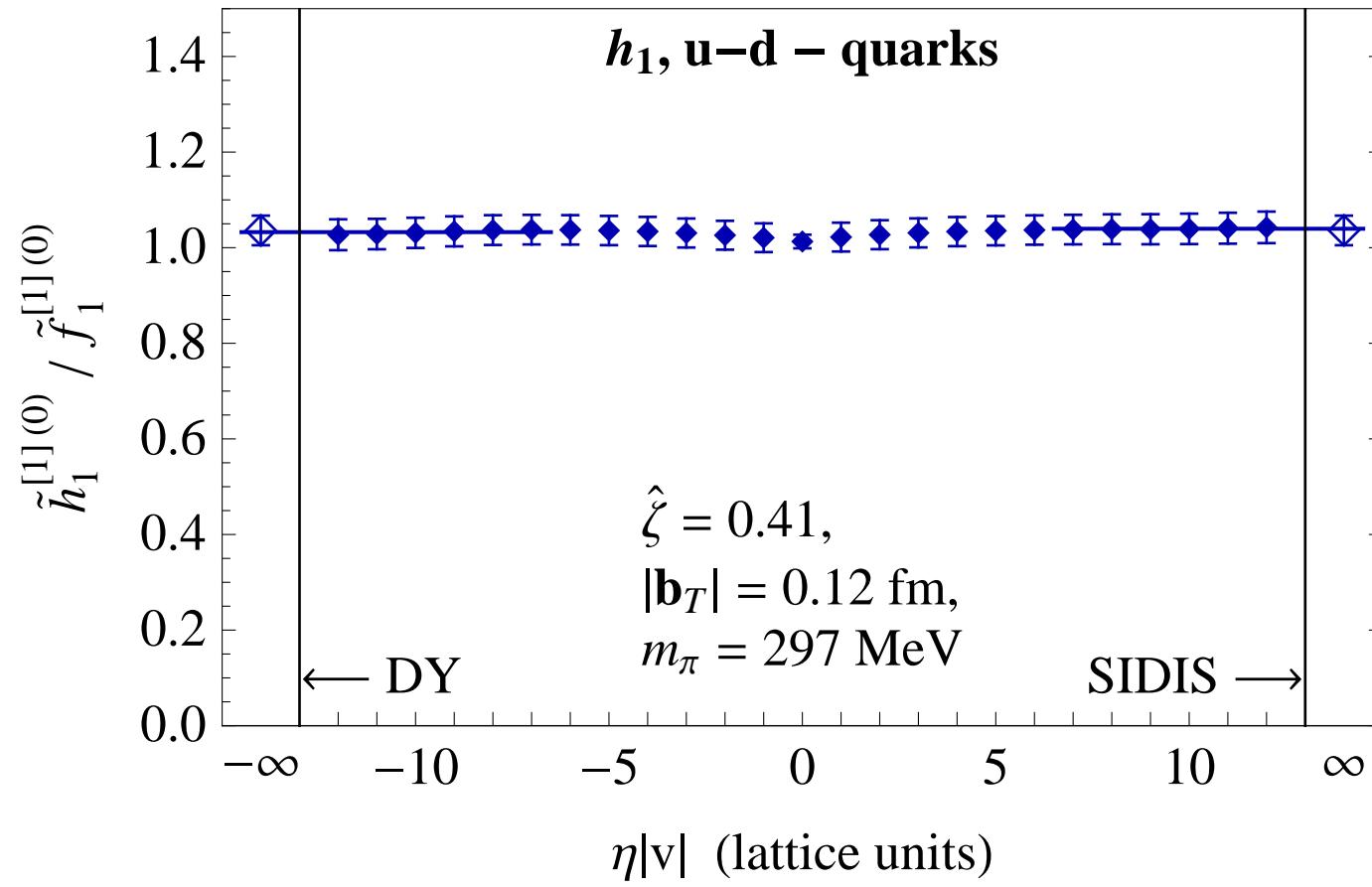
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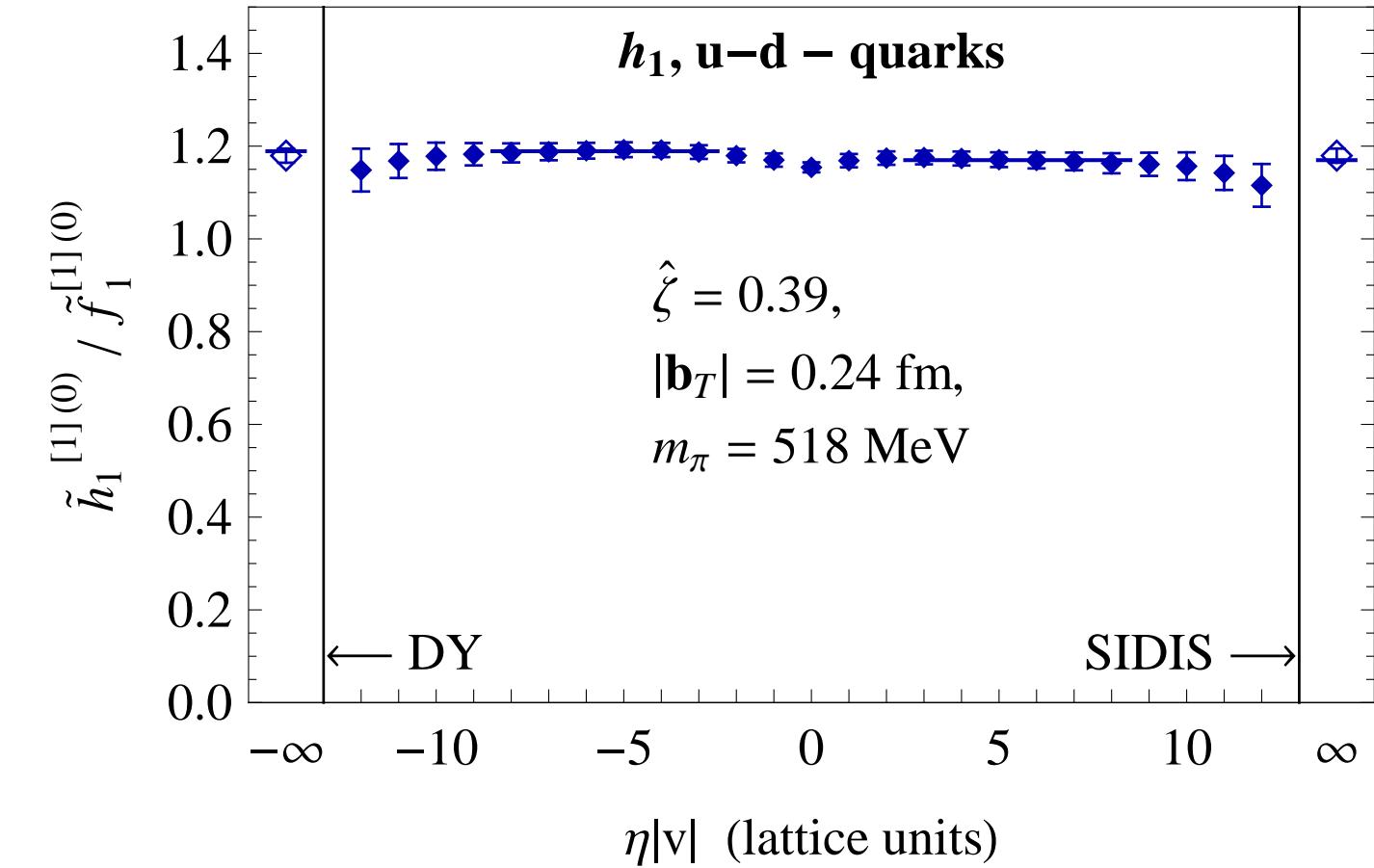
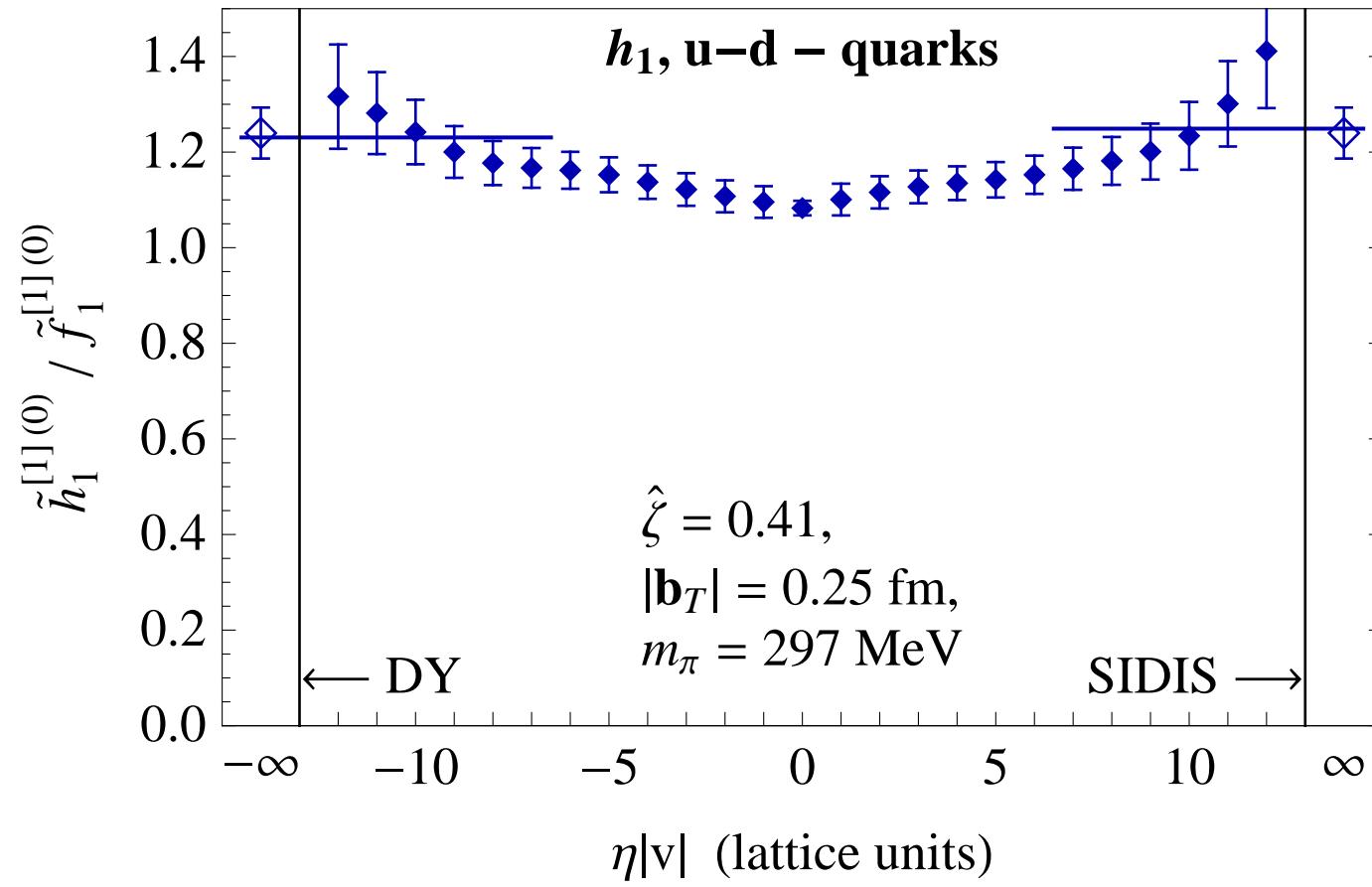
## Results: Transversity

Dependence on staple extent; sequence of panels at different  $|b_T|$



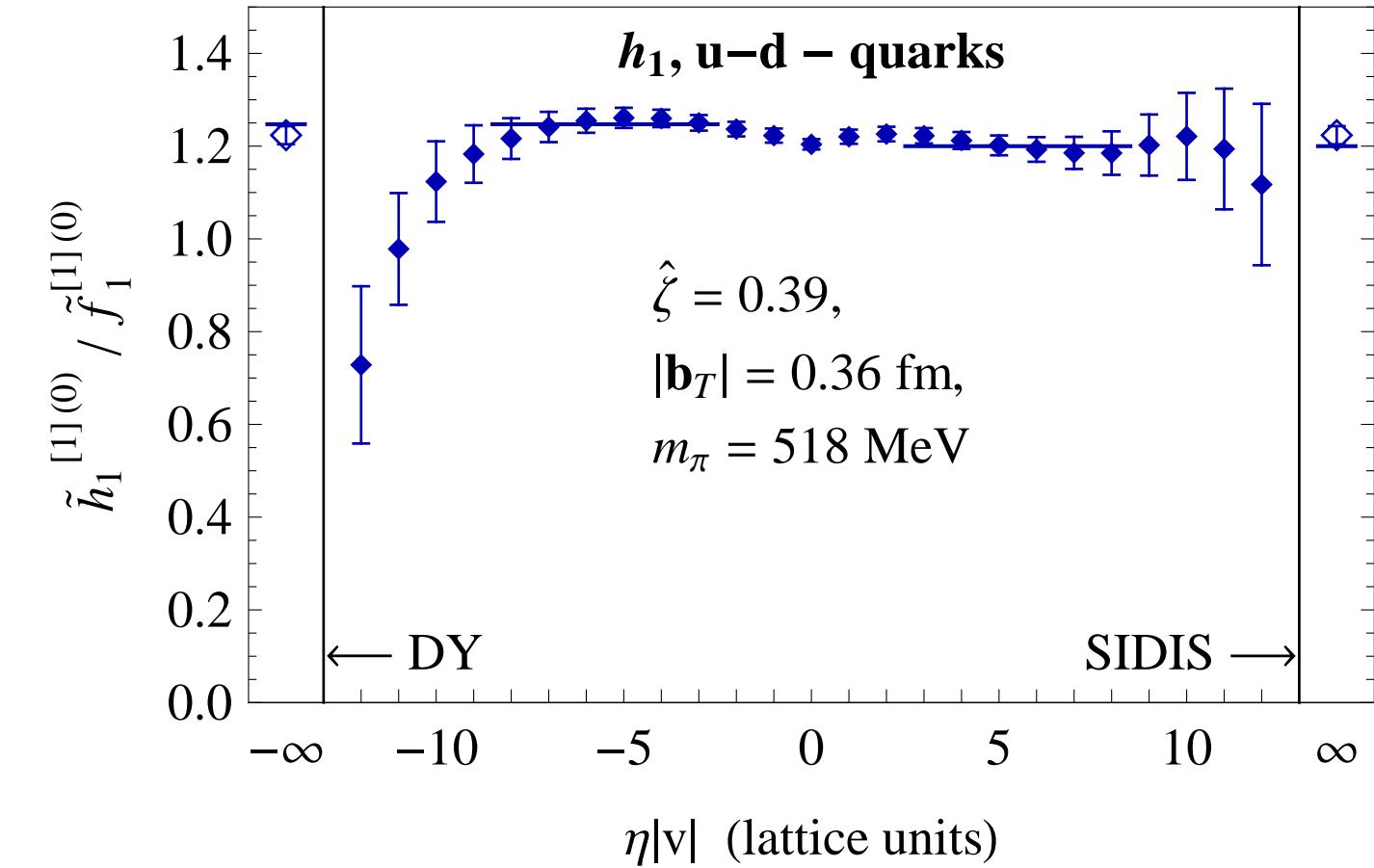
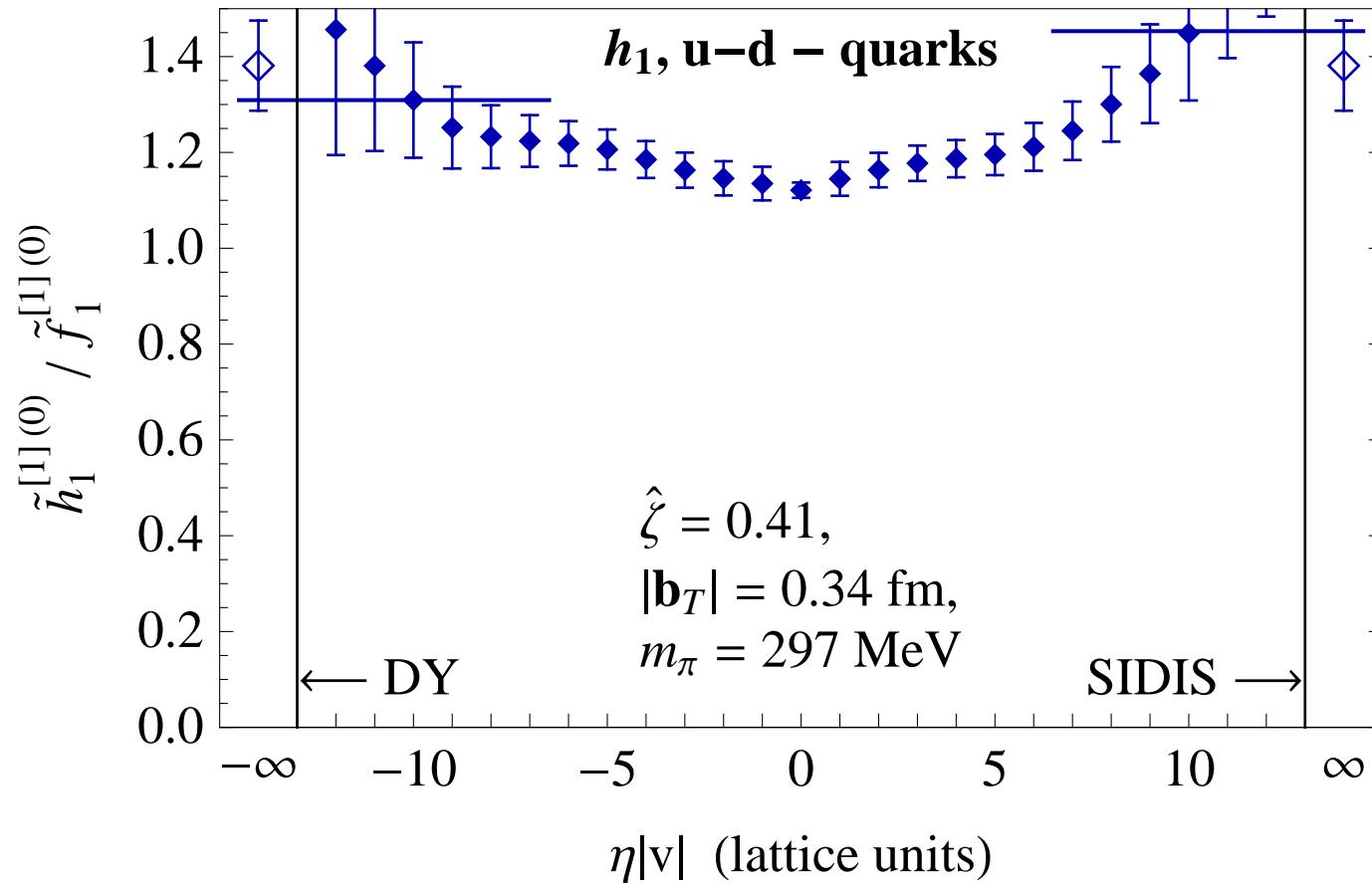
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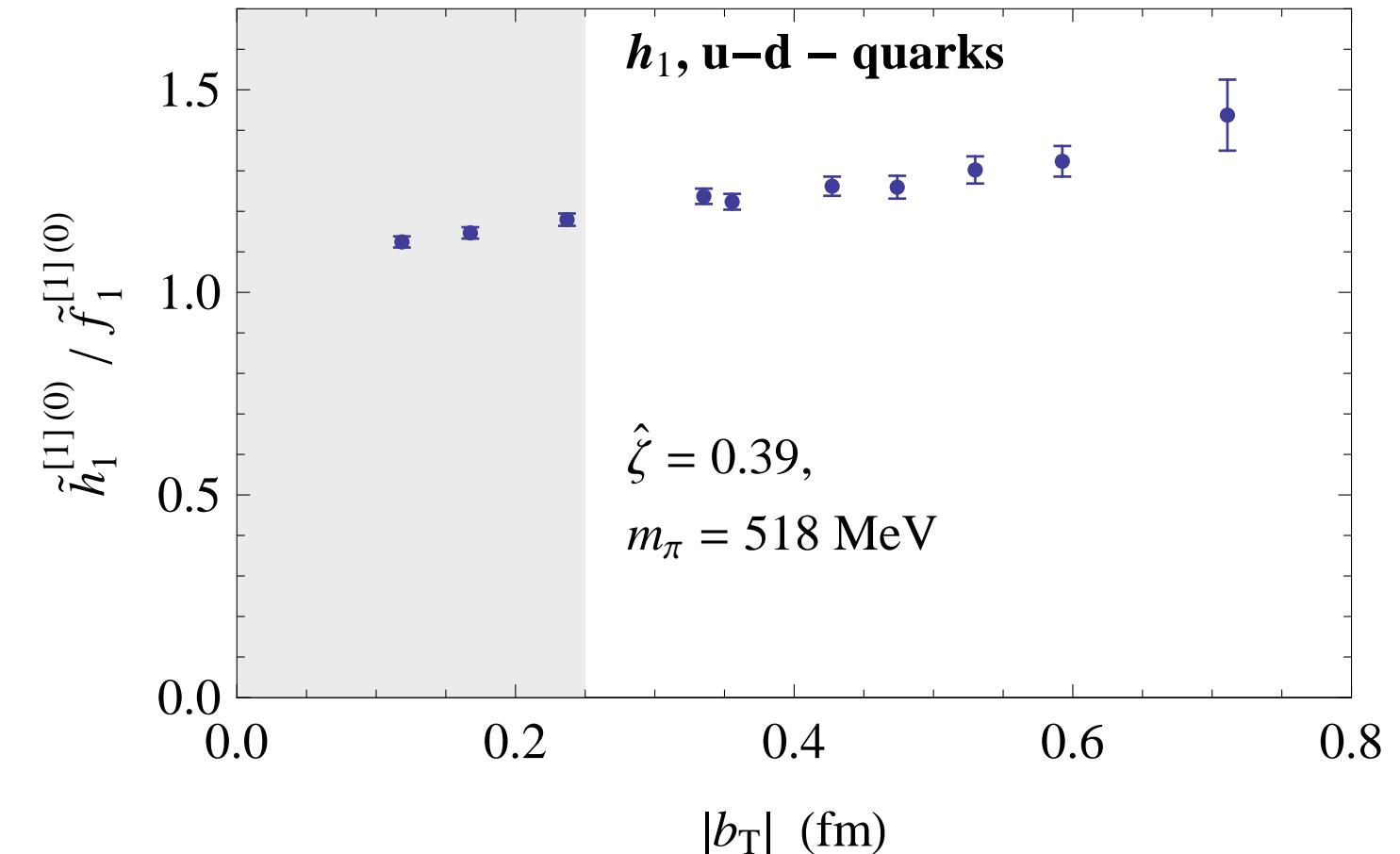
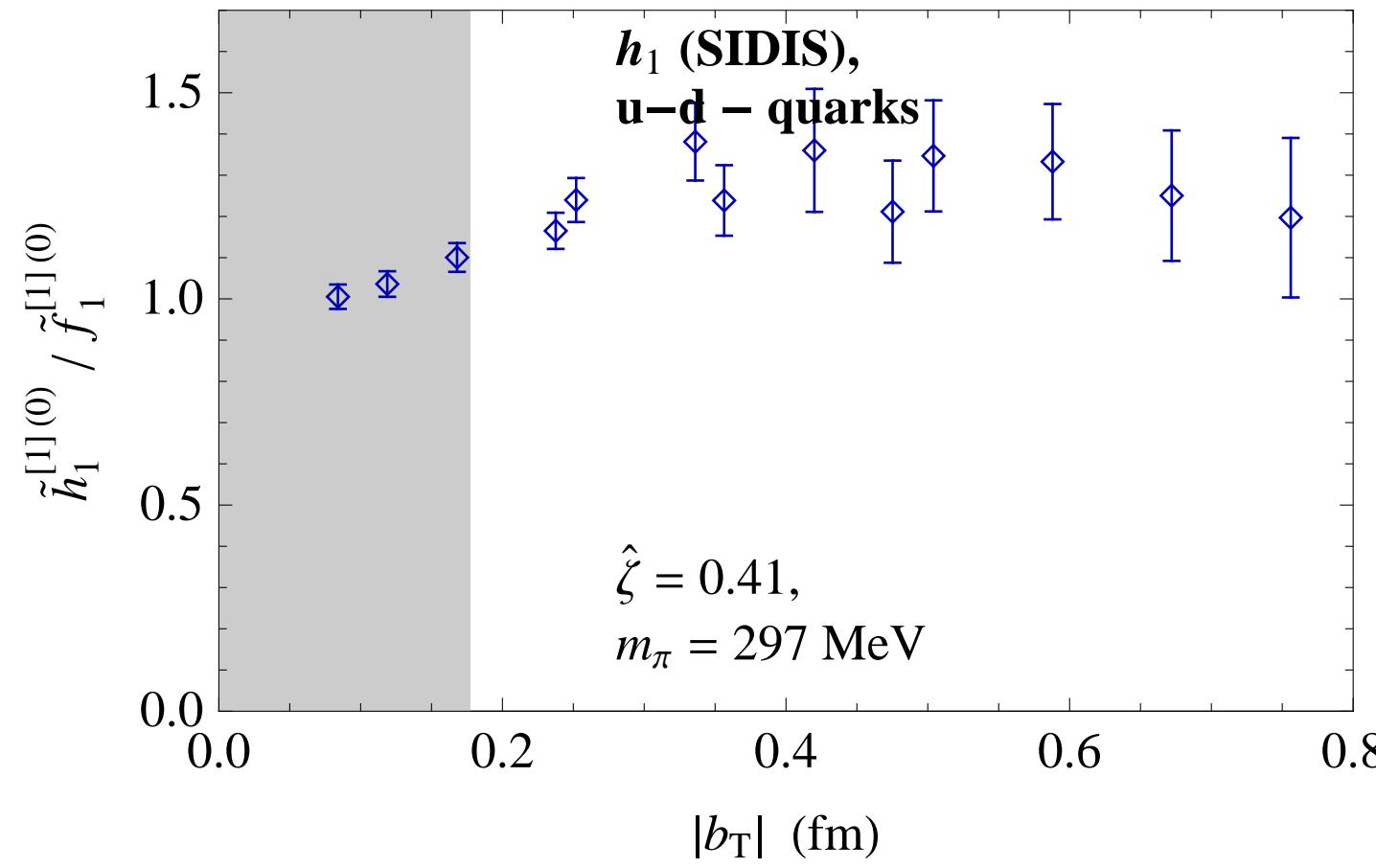
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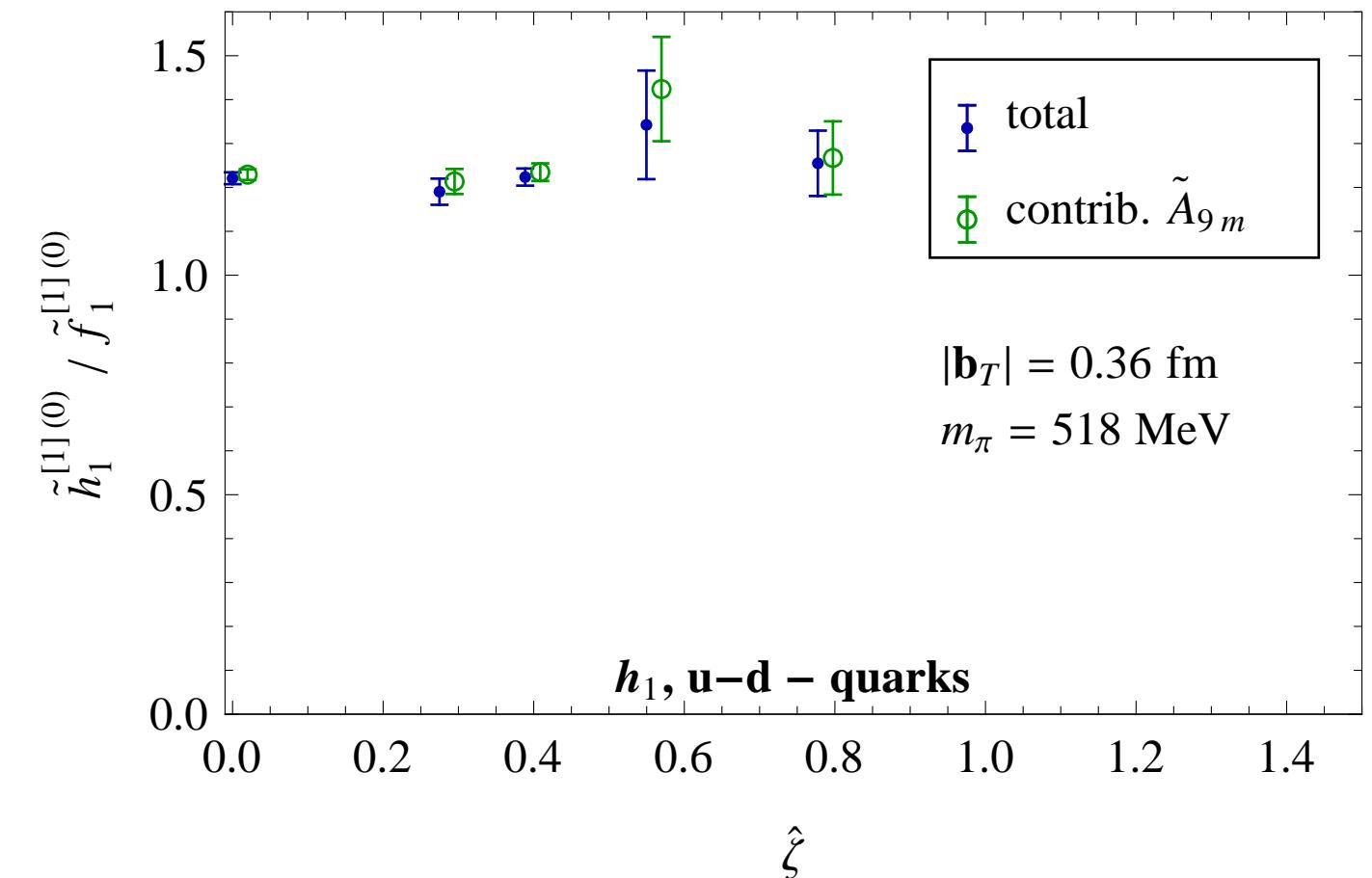
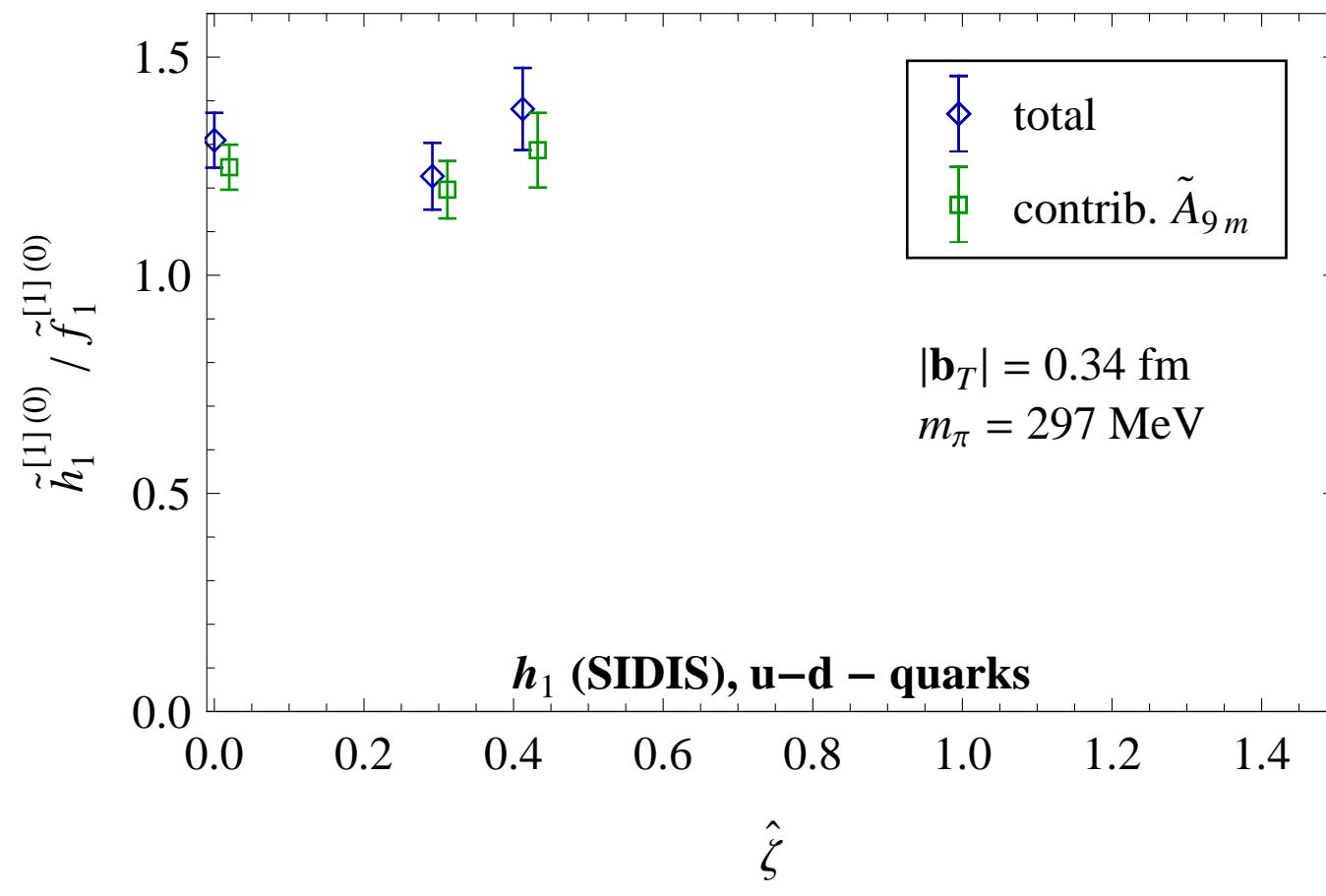
## Results: Transversity

Dependence of SIDIS/DY limit on  $|b_T|$



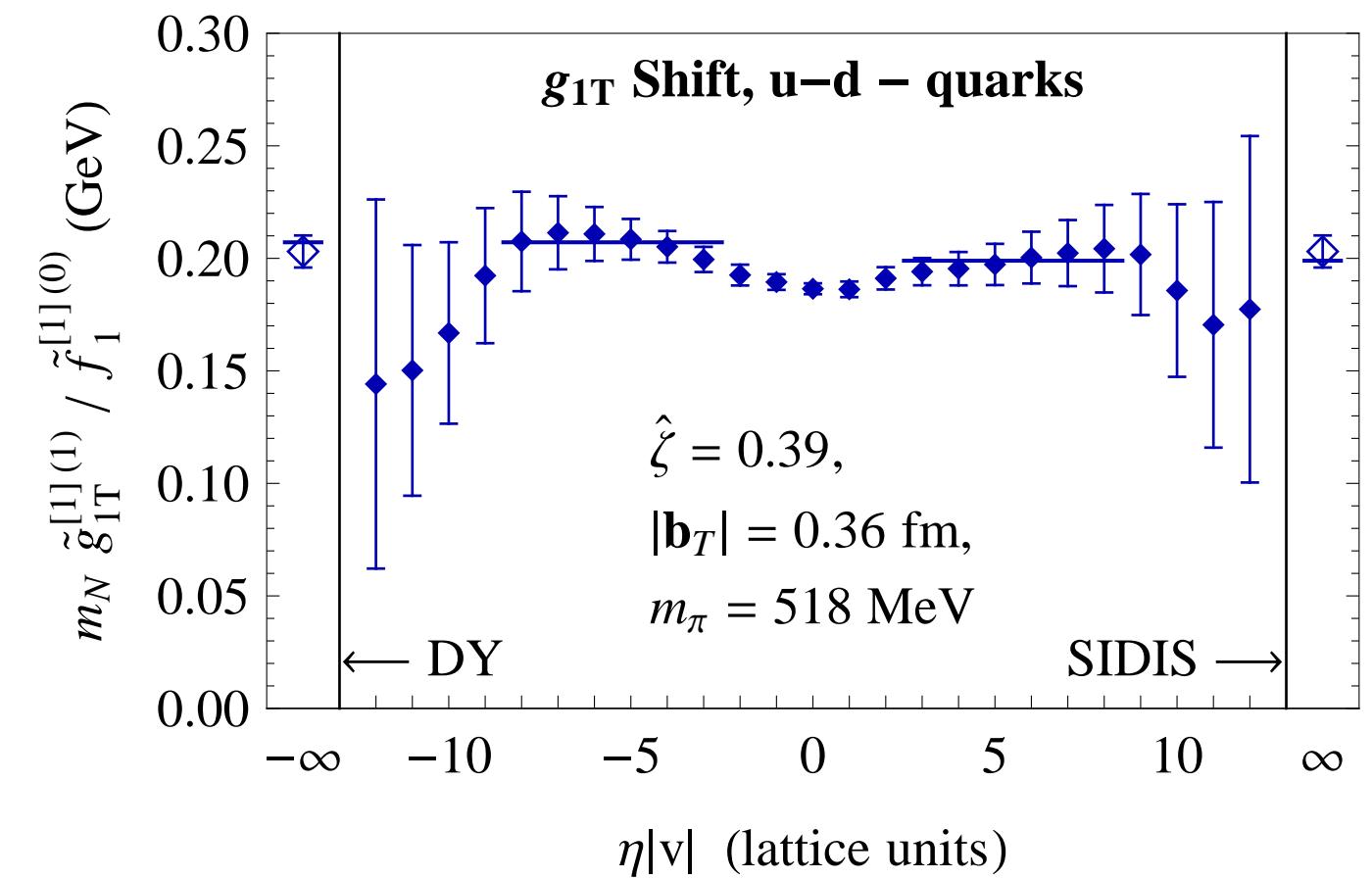
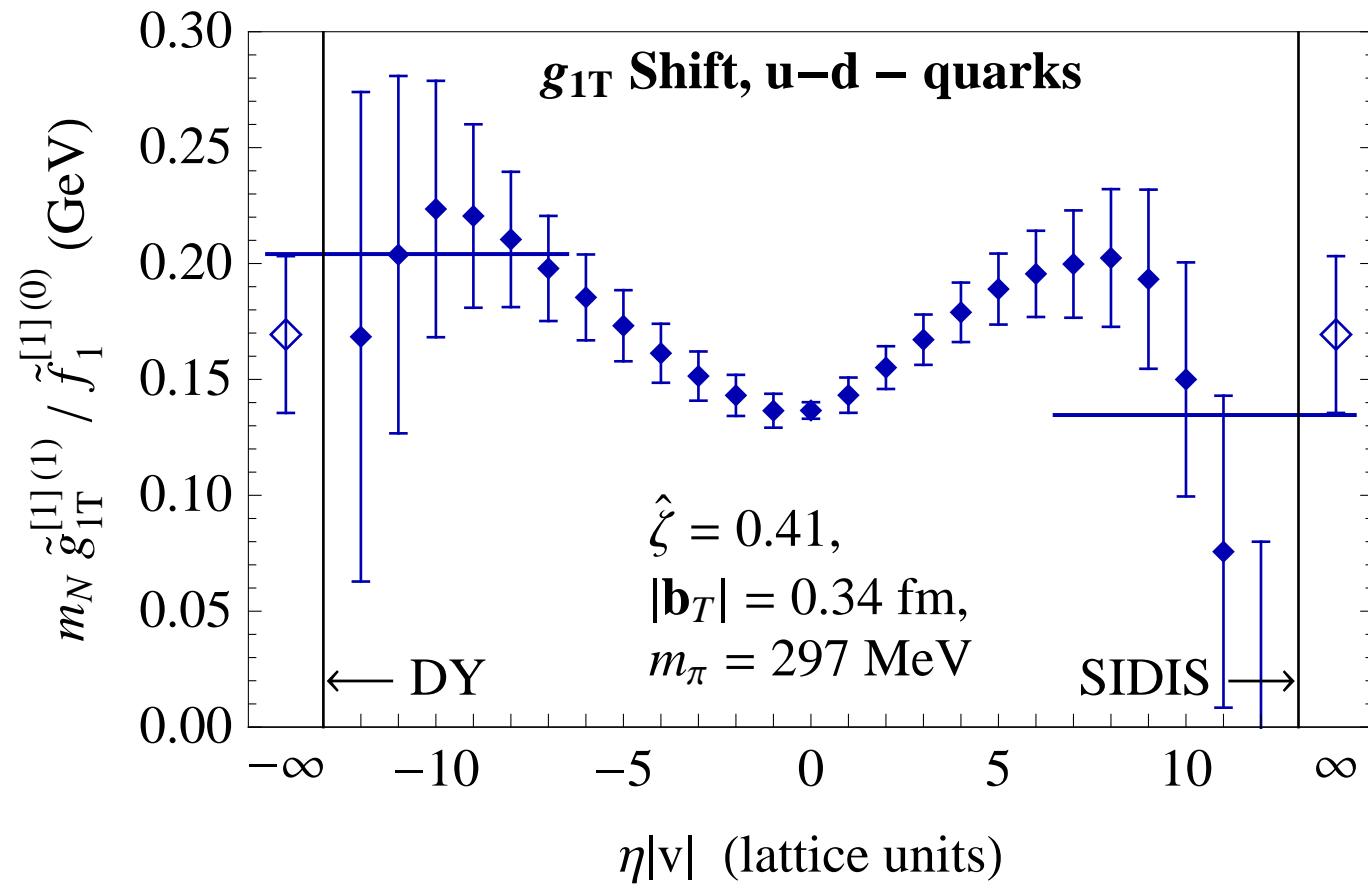
## Results: Transversity

Dependence of SIDIS/DY limit on  $\hat{\zeta}$



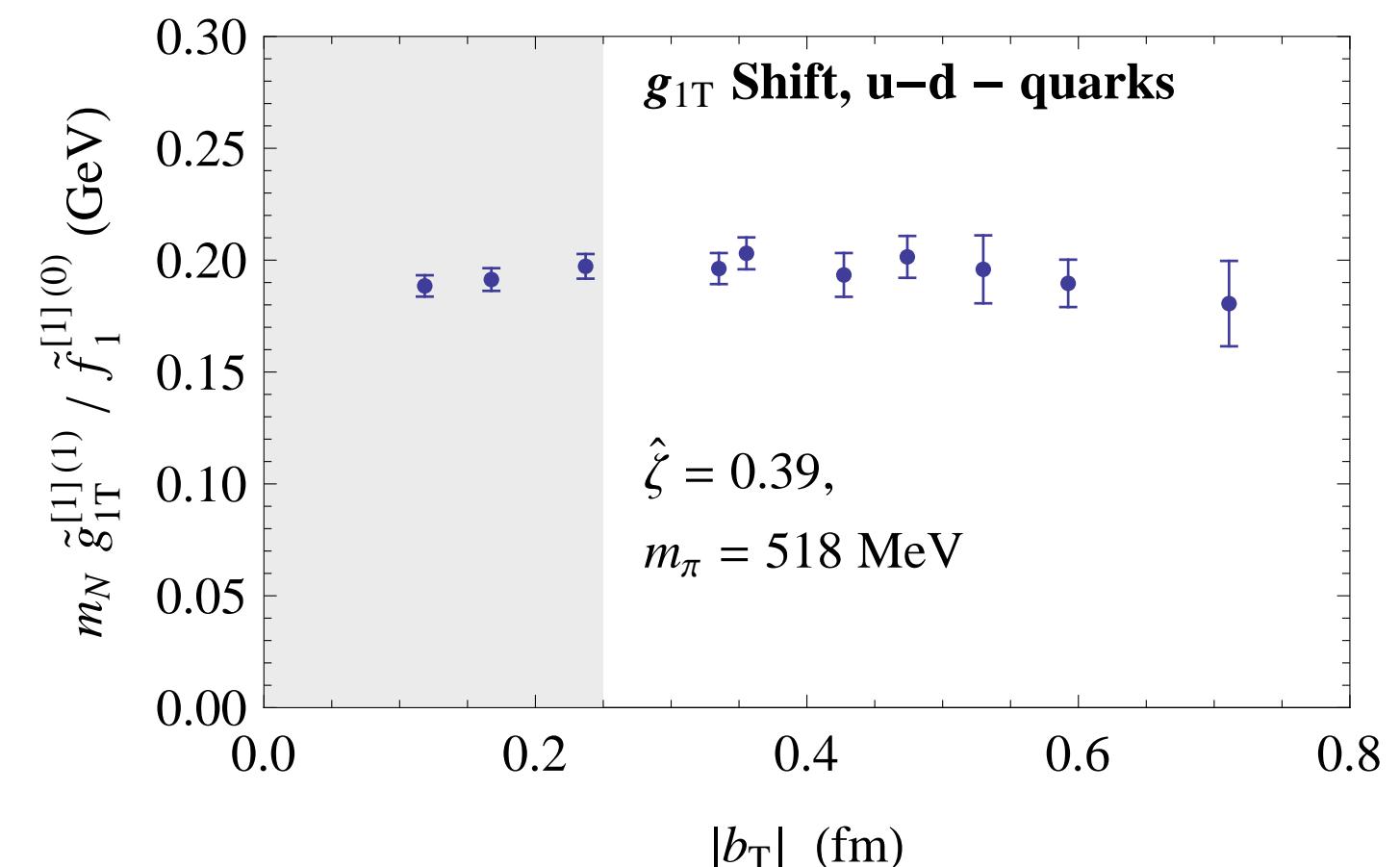
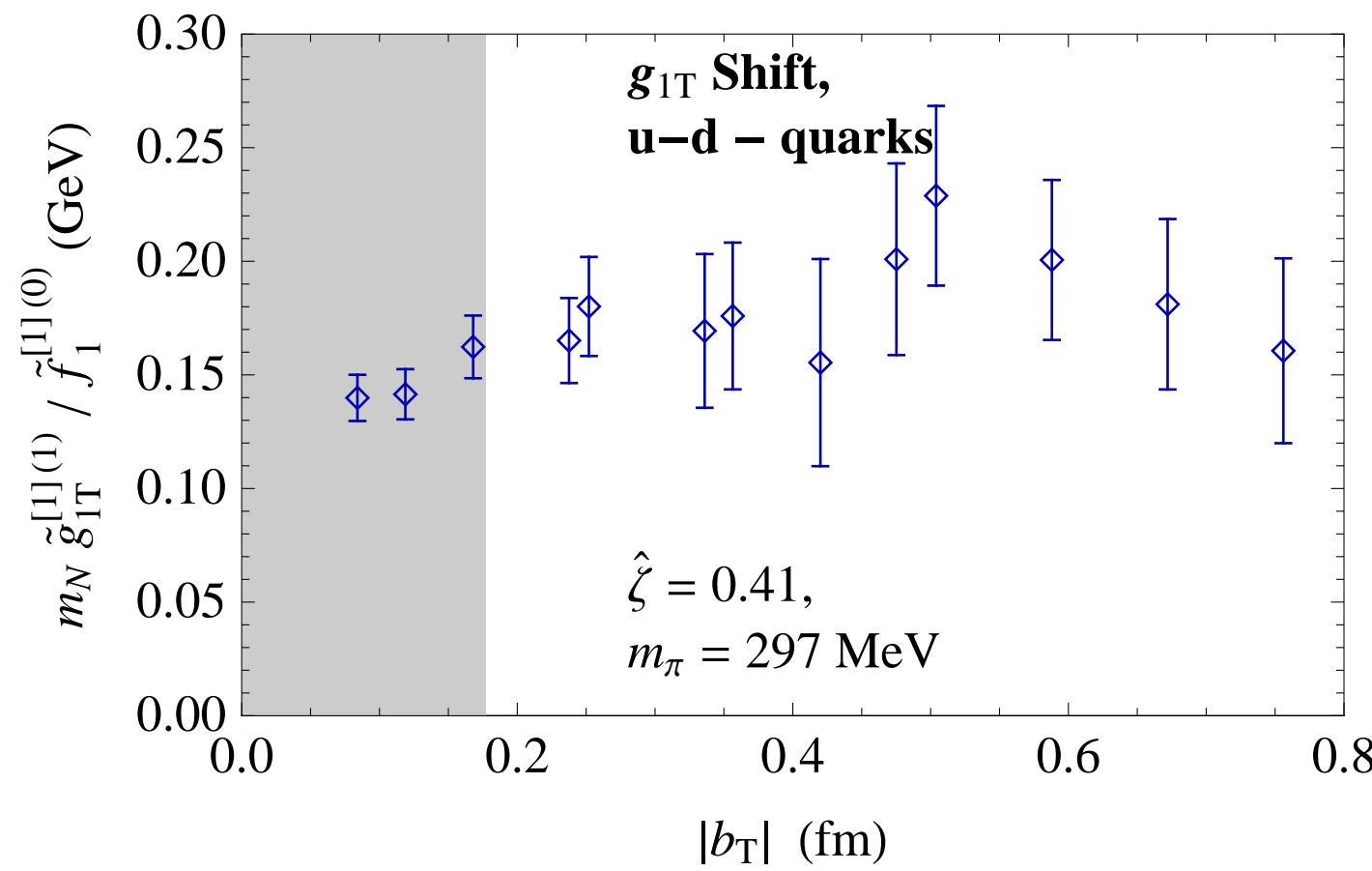
## Results: $g_{1T}$ worm gear shift

Dependence on staple extent



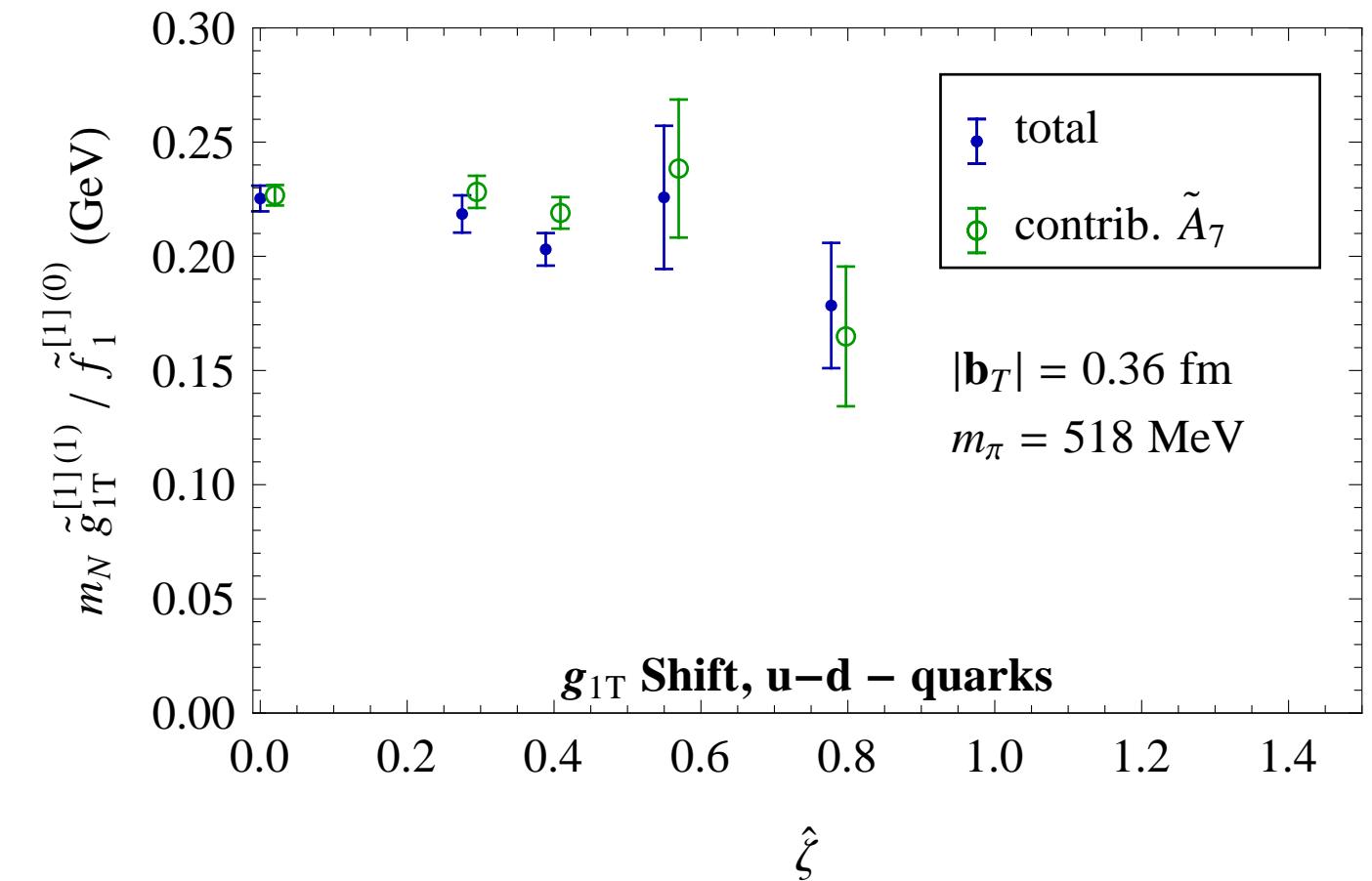
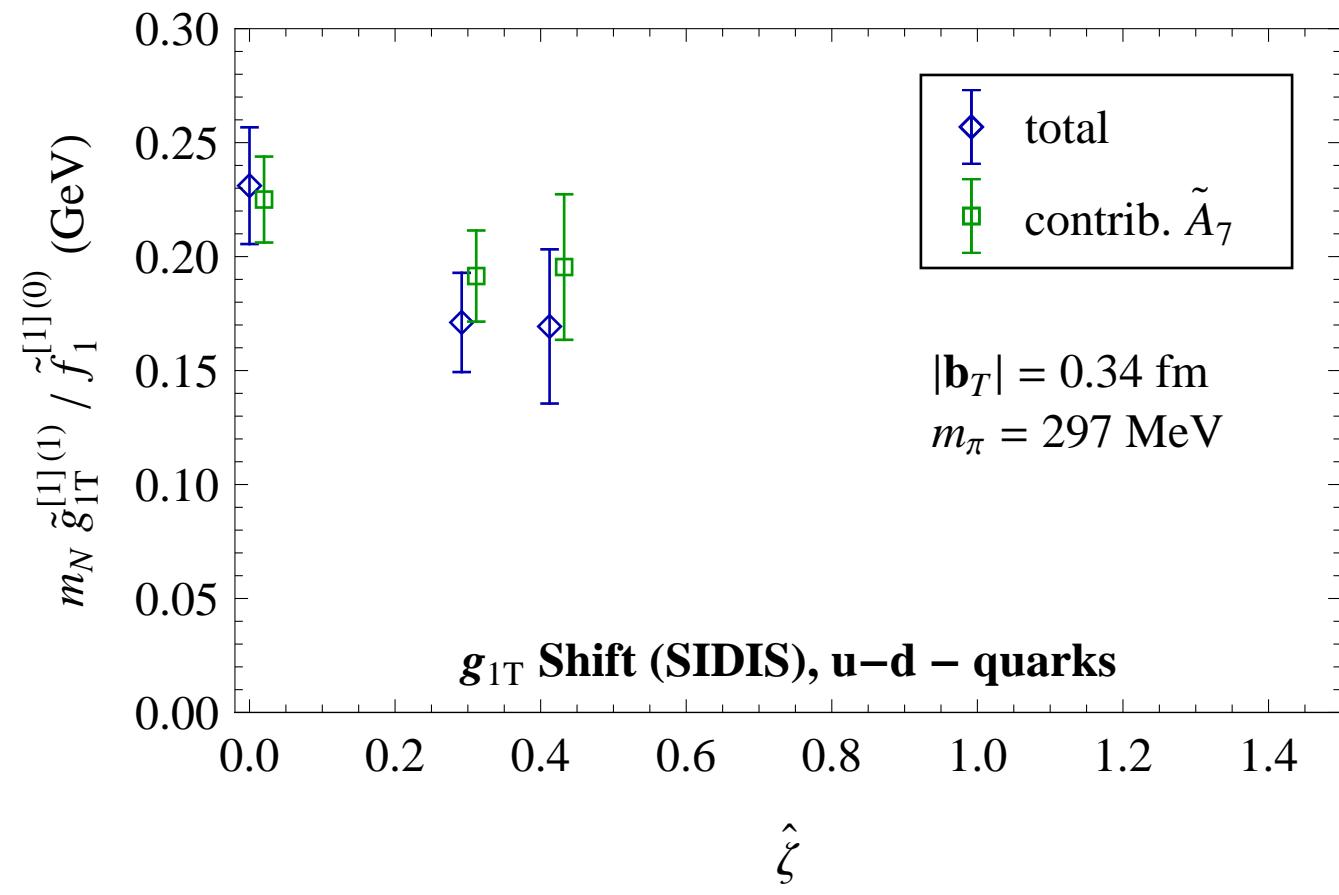
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Dependence of SIDIS/DY limit on  $|b_T|$



## Results: $g_{1T}$ worm gear shift

Dependence of SIDIS/DY limit on  $\hat{\zeta}$



## Conclusions and Outlook

- Continued exploration of TMDs using bilocal quark operators with staple-shaped gauge link structures; focus on lattice spacing effects, lighter pion mass.
- To avoid soft factors, multiplicative renormalization constants, considered appropriate ratios of Fourier-transformed TMDs (“shifts”).
- These observables show no statistically significant variation under the considered change of action, lattice spacing and pion mass, except at very short distances.
- In preparation: Comparison with USQCD clover ensemble at similar pion mass, to further test for universality and lattice spacing effects in isolation.
- Production underway on an RBC/UKQCD 180 MeV pion mass ensemble, production on an RBC/UKQCD ensemble at physical pion mass in preparation.
- Generalization to mixed transverse momentum / transverse position observables (Wigner functions) underway, to directly access quark orbital angular momentum.